非平稳异常噪声条件下的扩展目标跟踪方法

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摘 要：针对非平稳异常噪声环境下扩展目标跟踪问题，该文提出一种基于高斯-学生t混合(GSTM)扩展目标跟踪方法。首先，将过程噪声和量测噪声建模为GSTM分布，以表征非平稳厚尾噪声，并通过引入伯努利随机变量，将目标的运动状态和量测似然函数建模为分层高斯形式。其次，在随机矩阵(RMM)滤波框架下，使用变分贝叶斯方法详细推导了非平稳厚尾噪声下的GSTM扩展目标跟踪算法。该算法通过建模高斯噪声与厚尾噪声之间的非平稳过程，精确表征噪声特性，从而在非平稳异常噪声环境下稳健捕捉扩展目标的质心位置和轮廓形态。最后，构建非平稳异常噪声环境下的扩展目标跟踪仿真实验，并通过高斯-瓦瑟斯坦距离对实验结果进行效果评估，验证了所提出算法的合理性。此外，真实场景实验结果进一步证明了该算法在实际应用中的有效性和鲁棒性。

关键词：扩展目标跟踪；随机矩阵；高斯-学生t混合分布；变分贝叶斯方法

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1 引言

传统目标跟踪中，目标通常被简化为可以忽略空间形态的点目标[1,2]，也就是说被跟踪的目标在传感器中仅占据1个分辨率单元。然而，随着传感器分辨率的逐步提高，目标可以占据多个分辨率单元，即每个目标在每个时刻可以对应多个分辨率单元，这类目标被称为扩展目标 (Extended Target, ET)[3,4]。与仅估计运动状态的点目标跟踪不同，扩展目标跟踪(Extended Target Tracking, ETT)利用多个量测的空间分布信息，同时联合估计目标的运动和形状信息，因此在实际应用中越来越受关注。

目前，对于扩展目标形态建模方式分为基本几何形和不规则形。Koch[5]在2008年提出随机矩阵模型 (Random Matrix Model, RMM)，将目标形态建模为椭圆，并用对称正定的随机矩阵来表征。然而，RMM在跟踪过程中忽略了传感器噪声，当传感器噪声过大时，会降低形态的跟踪的精度。Feldmann等人[6]和Lan等人[7]对RMM方法进行改进，考虑噪声影响修正了量测似然函数。然而，该方法在估计过程中含有加性噪声项，无法得到递归解析解，文献[8]引入随机矩阵$\boldsymbol{B}_k$，用于描述目标观测形状相对于真实扩展状态在大小、形状及方向上的畸变。文献[9–11]提出一种乘性误差模型(Multiplicative Error Model, MEM)，该模型将形态建模为包含朝向和长短轴的3维向量，从而对椭圆目标进行参数化表示。为跟踪不规则形扩展目标，德国学者Baum等人[12]提出随机超曲面模型(Random Hypersurface Model, RHM)。RHM模型利用傅里叶级数展开的径向函数描述目标的星凸型不规则形状。而进一步地，Wahlström等人[13]又提出用高斯过程(Gaussian Process, GP)描述目标的边界函数。由于RHM和GP方法在描绘目标轮廓时需较多的量测信息，而对于稀疏量测下，RMM在跟踪中的适应性更好，所以其有更为广泛的应用前景。

现阶段的扩展目标跟踪研究多以高斯白噪声为前提，但实际中厚尾噪声更符合噪声分布特性。当传感器受到环境中的电磁干扰等异常状况时，会导致测量结果偏离正常值，形成比高斯噪声更重的尾部[14]。在这种更符合实际工况的异常噪声情况下，过程与量测噪声呈现重尾分布，便不再满足卡尔曼滤波器递推的高斯分布的前提，从而影响滤波器的准确性[15]。为了应对这个问题，一些学者已经开始研究各类型的鲁棒卡尔曼滤波器(Robust Kalman Filters, RKFs)。例如，M估计滤波器[16]、最大相

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关熵滤波器[17]和最小误差熵滤波器[18,19]，这些滤波器在一定程度上可以抵御厚尾噪声对状态估计器的影响，但在滤波过程没有利用其重尾特征，使其估计精度受到限制。此外，很多学者提出了将厚尾噪声建模为Student's t分布的方法，分别开发了学生t滤波器[20]和鲁棒学生t滤波器[21]。这些滤波器在平稳厚尾噪声环境中表现良好，但如果存在非平稳厚尾噪声，即噪声有时是高斯噪声，有时是参数不同的厚尾噪声，上述滤波器的性能便有所下降。于是，文献[22,23]提出了高斯-学生t混合分布(Gaussian-Student's t Mixture distribution, GSTM)，以适应非平稳噪声环境。该算法通过混合概率描绘噪声的非平稳性，并且通过设定混合概率值将其转化为单一的高斯噪声或厚尾噪声，从而适应更加复杂的滤波环境。综上所述，GSTM分布可以克服学生t滤波器在高斯环境下跟踪效果较差的缺点，从而在非平稳厚尾环境下对目标实现精确跟踪。所以，针对复杂环境下的扩展目标跟踪问题，需要提出一种量测噪声和过程噪声呈非平稳厚尾特性时，可以精确跟踪目标运动状态和形状的滤波器。

有鉴于此，本文针对非平稳过程噪声和量测噪声环境下扩展目标的跟踪问题，将噪声建模为GSTM分布，通过引入伯努利随机变量，将其表示为分层高斯形式，并将目标的扩展状态建模为逆威沙特分布，在变分贝叶斯(Variational Bayes, VB)的方法下得到目标的运动状态和扩展状态。最后通过仿真实验，验证本算法的有效性。

2 系统建模

相较于点目标模型，扩展目标的动态模型不仅描绘了目标质心的运动状态，还包括目标形状的演化。假设k时刻扩展目标状态可用集合$\boldsymbol{\zeta}_k$描述，即

$$\boldsymbol{\zeta}_k = (\boldsymbol{x}_k, \boldsymbol{X}_k) \tag{1}$$

其中，$\boldsymbol{x}_k$表示目标的运动状态，在2维空间中可用$\boldsymbol{x}_k = [x, y, \dot{x}, \dot{y}]^{\mathrm{T}}$表示，包含位置和速度信息；$\boldsymbol{X}_k$为对称正定矩阵，表示目标的扩展形态。

2.1 质心运动模型

通常情况下，k时刻的目标状态空间为

$$\boldsymbol{x}_k = (\boldsymbol{F}_{k|k-1} \otimes \boldsymbol{I}_d)\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1} \tag{2}$$

其中，$\boldsymbol{F}_{k|k-1}$表示目标运动的状态转移矩阵；$\boldsymbol{I}_d$表示d维的单位矩阵；$\otimes$表示克罗内克积；$\boldsymbol{w}_{k-1}$表示协方差为$\boldsymbol{Q}_{k-1} = \boldsymbol{D}_{k-1|k-2} \otimes \boldsymbol{X}_{k-1}$的过程噪声，$\boldsymbol{D}_{k-1|k-2}$为系数矩阵。

2.2 目标量测模型

假设每个量测均是由散布在扩展目标表面上的量测源产生，即量测$\boldsymbol{z}_k^j$源于被噪声$\boldsymbol{v}_k^j$破坏的测量源$\boldsymbol{y}_k^j$，量测可表示为

$$\boldsymbol{z}_k^j = (\boldsymbol{H}_k \otimes \boldsymbol{I}_d)\boldsymbol{x}_k + \boldsymbol{v}_k^j \tag{3}$$

其中，$\boldsymbol{y}_k^j = (\boldsymbol{H}_k \otimes \boldsymbol{I}_d)\boldsymbol{x}_k$表示量测源的位置，$\tilde{\boldsymbol{H}}_k = \boldsymbol{H}_k \otimes \boldsymbol{I}_d$；$\boldsymbol{v}_k^j$表示协方差为$s\boldsymbol{X}_k + \boldsymbol{R}_k$的量测噪声，$s$为缩放因子，一般取$s = 1/4$[6]。记$k$时刻目标所对应的全部量测集合为$\boldsymbol{Z}_k = \{\boldsymbol{z}_k^j\}_{j=1}^{n_k}$，$n_k$表示$k$时刻目标产生的量测数，一般服从均值为$p$的泊松分布[24]。

2.3 扩展状态模型

扩展目标的形状会随着时间演变，假设扩展状态的后验概率密度$p(\boldsymbol{X}_{k-1} \mid \boldsymbol{Z}^{k-1})$服从逆威沙特分布，即

$$p(\boldsymbol{X}_{k-1} \mid \boldsymbol{Z}^{k-1}) = \mathrm{IW}(\boldsymbol{X}_{k-1}; v_{k-1|k-1}, \boldsymbol{V}_{k-1|k-1}) \tag{4}$$

其中，$\boldsymbol{Z}^{k-1}$表示前$k-1$时刻的所有量测；$v_{k-1|k-1}$和$\boldsymbol{V}_{k-1|k-1}$分别为逆威沙特分布中的自由度参数和参数矩阵。

假设扩展状态$\boldsymbol{X}_k$的状态转移概率密度$p(\boldsymbol{X}_k|\boldsymbol{X}_{k-1})$服从威沙特分布，即

$$p(\boldsymbol{X}_k \mid \boldsymbol{X}_{k-1}) = W\left(\boldsymbol{X}_k; \delta_{k|k-1}, \boldsymbol{X}_{k-1}/\delta_{k|k-1}\right) \tag{5}$$

其中，$\delta_{k|k-1}$表示自由度参数，用于描述扩展目标的形状大小变化。

2.4 异常噪声模型

在实际工程应用中，由于环境变化，状态异常值和量测异常值可能出现在不同时间段，从而产生非平稳的重尾过程噪声和测量噪声[22]。例如，扩展目标在遮挡物较多的复杂场景中运动时，目标的回波信号可能受到大量异常噪声的干扰，此时可以建模为厚尾噪声；当目标又进入空旷环境中，信号的异常值会显著减少，则可以建模为高斯噪声。为灵活适应不同的噪声分布，可以引入GSTM分布，该分布能够灵活调整以适应不同的噪声分布。满足GSTM分布的随机变量$\boldsymbol{r}$的概率密度函数可表示为

$$p(\boldsymbol{r}|\varsigma) = \varsigma N(\boldsymbol{r}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) + (1-\varsigma)\mathrm{St}(\boldsymbol{r}; \boldsymbol{\mu}, \boldsymbol{\Sigma}, \nu) \tag{6}$$

其中，$\boldsymbol{\mu}$, $\boldsymbol{\Sigma}$和ν分别为均值向量、尺度矩阵和自由度参数；混合概率ς服从贝塔分布，可表示为

$$p(\varsigma) = \mathrm{Be}(\varsigma; l, 1-l) \tag{7}$$

其中，l表示先验形状参数。

利用式(6)和式(7)，GSTM分布的随机向量$\boldsymbol{r}$的概率密度函数可表示为

$$p(\boldsymbol{r}) = \int p(\boldsymbol{r}|\varsigma)p(\varsigma)\mathrm{d}\varsigma = lN(\boldsymbol{r};\boldsymbol{\mu},\boldsymbol{\Sigma}) + (1-l)\mathrm{St}(\boldsymbol{r};\boldsymbol{\mu},\boldsymbol{\Sigma},\nu) \tag{8}$$

由式(8)可以清楚地看到，当先验混合概率分别取$l=1$或$l=0$时，所提出的GSTM分布退化为高斯分布或Student's t分布。

引入隐变量ε，式(8)可表示为

$$p(\boldsymbol{r}|\varepsilon) = lN(\boldsymbol{r};\boldsymbol{\mu},\boldsymbol{\Sigma}) + (1-l)N(\boldsymbol{r};\boldsymbol{\mu},\boldsymbol{\Sigma}/\varepsilon) \tag{9}$$

其中，隐变量ε满足式(10)

$$p(\varepsilon|\nu) = G(\varepsilon;\nu/2,\nu/2) \tag{10}$$

其中，$G(x;\kappa,\varpi) = \dfrac{\varpi^{\kappa}}{\Gamma(\kappa)}x^{\kappa-1}\mathrm{e}^{-\varpi x}$表示形状参数和尺度参数分别$\kappa$为和$\varpi$的伽马分布。由于固定学生t分布中的自由度参数$\nu$会对噪声的建模造成误差，因此，将自由度参数$\nu$建模为伽马函数。

在VB框架下，需要对混合概率进行在线推断，通过伯努利变量的引入，实现模型中混合概率的更新。此外，通过引入伯努利变量c，将混合概率建模为二项分布，使得ς的似然函数表示为伯努利分布，即

$$p(c|\varsigma) = \varsigma^{c}(1-\varsigma)^{1-c} \tag{11}$$

最后，提出的GSTM的概率密度函数可以用以下分层高斯形式表示

$$p(\boldsymbol{r}|c,\varepsilon) = [N(\boldsymbol{r};\boldsymbol{\mu},\boldsymbol{\Sigma})]^{c}[N(\boldsymbol{r};\boldsymbol{\mu},\boldsymbol{\Sigma}/\varepsilon)]^{(1-c)} \tag{12}$$

当过程噪声和量测噪声为非平稳噪声时，可将运动状态一步预测函数和量测似然函数写成

$$\left.\begin{aligned}
&p(\boldsymbol{x}_k|\boldsymbol{X}_k,\boldsymbol{Z}^{k-1},\sigma_k,t_k)\\
&\quad=\left[\ N(\boldsymbol{x}_k;\boldsymbol{x}_{k|k-1},\boldsymbol{P}_{k|k-1}\otimes\boldsymbol{X}_k)\ \right]^{t_k}\\
&\qquad\cdot\left[\ N(\boldsymbol{x}_k;\boldsymbol{x}_{k|k-1},\left(\boldsymbol{P}_{k|k-1}\otimes\boldsymbol{X}_k\right)/\sigma_k)\ \right]^{(1-t_k)}\\
&p(\sigma_k) = G\left(\sigma_k;\frac{\omega_k}{2},\frac{\omega_k}{2}\right)\\
&p(t_k|\tau_k) = \tau_k^{t_k}(1-\tau_k)^{(1-t_k)}\\
&p(\tau_k) = \mathrm{Be}(\tau_k;q_0,1-q_0)
\end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned}
&p(\boldsymbol{Z}_k|\boldsymbol{x}_k,\boldsymbol{X}_k,n_k,\rho_k,\gamma_k)\\
&\quad=\prod_{j=1}^{n_k}\left[N(\boldsymbol{z}_k^j;\tilde{\boldsymbol{H}}_k\boldsymbol{x}_k,s\boldsymbol{X}_k+\boldsymbol{R}_k)\right]^{\gamma_k}\\
&\qquad\cdot\left[N(\boldsymbol{z}_k^j;\tilde{\boldsymbol{H}}_k\boldsymbol{x}_k,(s\boldsymbol{X}_k+\boldsymbol{R}_k)/\rho_k)\right]^{(1-\gamma_k)}\\
&p(\rho_k) = G\left(\rho_k;\frac{u_k}{2},\frac{u_k}{2}\right)\\
&p(\gamma_k|\pi_k) = \pi_k^{\gamma_k}(1-\pi_k)^{(1-\gamma_k)}\\
&p(\pi_k) = \mathrm{Be}(\pi_k;g_0,1-g_0)
\end{aligned}\right\} \tag{14}$$

其中，t_k和γ_k为伯努利变量，σ_k和ρ_k为隐变量，τ_k和π_k为混合概率，ω_k和u_k为自由度参数，q_0和g_0为贝塔分布的先验形状参数。

3　滤波算法

3.1　时间更新

在贝叶斯滤波框架下，目标的运动状态和扩展状态可以分开讨论，即$\boldsymbol{x}_k$和$\boldsymbol{X}_k$的联合预测密度可表示为

$$p(\boldsymbol{x}_k,\boldsymbol{X}_k\mid\boldsymbol{Z}^{k-1}) = p(\boldsymbol{x}_k\mid\boldsymbol{X}_k,\boldsymbol{Z}^{k-1})p(\boldsymbol{X}_k\mid\boldsymbol{Z}^{k-1}) \tag{15}$$

式(13)中的均值向量和尺度矩阵可表示为

$$\boldsymbol{x}_{k|k-1} = (\boldsymbol{F}_{k|k-1}\otimes\boldsymbol{I}_d)\boldsymbol{x}_{k-1|k-1} \tag{16}$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_{k|k-1}\boldsymbol{P}_{k-1|k-1}\boldsymbol{F}_{k|k-1}^{\mathrm{T}} + \boldsymbol{D}_{k|k-1} \tag{17}$$

扩展状态的预测密度可表示为

$$\begin{aligned}
p(\boldsymbol{X}_k\mid\boldsymbol{Z}^{k-1}) = &\int p\left(\boldsymbol{X}_k|\boldsymbol{X}_{k-1}\right)\\
&\cdot p\left(\boldsymbol{X}_{k-1}|\boldsymbol{Z}^{k-1}\right)\mathrm{d}\boldsymbol{X}_{k-1}\\
=&\ \mathrm{IW}(\boldsymbol{X}_k;v_{k|k-1},\boldsymbol{V}_{k|k-1})
\end{aligned} \tag{18}$$

其中

$$v_{k|k-1} = \mathrm{e}^{-\Delta t_k/\tau}v_{k-1|k-1} \tag{19}$$

$$\boldsymbol{V}_{k|k-1} = \frac{\mathrm{e}^{-\Delta t_k/\tau}v_{k-1|k-1}-d-1}{v_{k-1|k-1}-d-1}\boldsymbol{V}_{k-1|k-1} \tag{20}$$

其中，$\triangle\, t_k = t_k - t_{k-1}$为时间间隔，$\tau$为衰减系数。

根据式(13)、式(15)和式(18)，可将联合预测概率密度函数$p(\boldsymbol{x}_k,\boldsymbol{X}_k\mid\boldsymbol{Z}^{k-1},\sigma_k,t_k)$写成

$$\begin{aligned}
&p(\boldsymbol{x}_k,\boldsymbol{X}_k\mid\boldsymbol{Z}^{k-1},\sigma_k,t_k)\\
&\quad=\left[\ N(\boldsymbol{x}_k;\boldsymbol{x}_{k|k-1},\boldsymbol{P}_{k|k-1}\otimes\boldsymbol{X}_k)\ \right]^{t_k}\\
&\qquad\cdot\mathrm{IW}(\boldsymbol{X}_k;v_{k|k-1},\boldsymbol{V}_{k|k-1})\\
&\qquad\cdot\left[\ N(\boldsymbol{x}_k;\boldsymbol{x}_{k|k-1},\left(\boldsymbol{P}_{k|k-1}\otimes\boldsymbol{X}_k\right)/\sigma_k)\ \right]^{(1-t_k)}
\end{aligned} \tag{21}$$

3.2　似然函数

给定目标的运动状态和扩展状态以及量测次数下的似然函数可更新为

$$\begin{aligned}
&p(\boldsymbol{Z}_k|\boldsymbol{x}_k,\boldsymbol{X}_k,n_k,\rho_k,\gamma_k)\\
&\quad=\left[N(\boldsymbol{z}_k;\tilde{\boldsymbol{H}}_k\boldsymbol{x}_k,(s\boldsymbol{X}_k+\boldsymbol{R}_k)/n_k\right]^{\gamma_k}\\
&\qquad\cdot\left[\mathrm{LW}(\bar{\boldsymbol{Z}}^k;n_k-1,s\boldsymbol{X}_k+\boldsymbol{R}_k)\right]^{\gamma_k}\\
&\qquad\cdot\left[\mathrm{LW}(\bar{\boldsymbol{Z}}^k;n_k-1,(s\boldsymbol{X}_k+\boldsymbol{R}_k)/\rho_k)\right]^{(1-\gamma_k)}\\
&\qquad\cdot\left[N(\boldsymbol{z}_k;\tilde{\boldsymbol{H}}_k\boldsymbol{x}_k,(s\boldsymbol{X}_k+\boldsymbol{R}_k)/n_k\rho_k)\right]^{(1-\gamma_k)}
\end{aligned} \tag{22}$$

量测质心$\boldsymbol{z}_k$和散射矩阵$\bar{\boldsymbol{Z}}_k$可表示为

$$\boldsymbol{z}_k = \frac{1}{n_k}\sum_{j=1}^{n_k}\boldsymbol{z}_k^j \tag{23}$$

$$\overline{\boldsymbol{Z}}_k = \sum_{j=1}^{n_k}(\boldsymbol{z}_k^j - \boldsymbol{z}_k)(\boldsymbol{z}_k^j - \boldsymbol{z}_k)^{\mathrm{T}} \tag{24}$$

3.3 量测更新

由于所求状态向量与参数之间存在耦合关系，通过解析方法得到运动状态和扩展状态的联合后验分布很麻烦。因此可采取VB方法求得状态的近似解[25]，使用一个较为简单的分布$q(\boldsymbol{\Phi}_k)$近似等价于复杂的后验概率$p(\boldsymbol{\Phi}_k|\boldsymbol{Z}^k)$，通过最小化两者之间的K-L散度[26]，得到一个接近于真实后验分布的近似解。其中$\boldsymbol{\Phi}_k \triangleq \{\boldsymbol{x}_k, \boldsymbol{X}_k, \sigma_k, \rho_k, t_k, \gamma_k, \tau_k, \pi_k\}$。$q(\boldsymbol{\Phi}_k)$的最优解满足

$$\ln q(\varphi) = E_{\boldsymbol{\Phi}_k(-\varphi)}\left[\ln p(\boldsymbol{\Phi}_k, \boldsymbol{Z}^k)\right] + \mathrm{c}_\varphi \tag{25}$$

其中，φ为$\boldsymbol{\Phi}_k$中的参数；$\boldsymbol{\Phi}_k(-\varphi)$为$\boldsymbol{\Phi}_k$中除$\varphi$以外的参数；$\mathrm{c}_\varphi$表示独立于$\varphi$的常数。通过固定点迭代求解式(25)中的参数，即在第i次更新时，将$q(\boldsymbol{\Phi}_k(-\varphi))$固定为$q^{i-1}(\boldsymbol{\Phi}_k(-\varphi))$。其联合分布$p(\boldsymbol{\Phi}_k, \boldsymbol{Z}^k)$可表示为

$$\begin{aligned}
&p(\boldsymbol{\Phi}_k, \boldsymbol{Z}^k)\\
&= p(\boldsymbol{Z}^{k-1})\left[\ N(\boldsymbol{x}_k; \boldsymbol{x}_{k|k-1}, \boldsymbol{P}_{k|k-1}\otimes\boldsymbol{X}_k)\ \right]^{t_k}\\
&\cdot\left[\ N(\boldsymbol{x}_k; \boldsymbol{x}_{k|k-1}, (\boldsymbol{P}_{k|k-1}\otimes\boldsymbol{X}_k)/\sigma_k)\ \right]^{(1-t_k)}\\
&\cdot \mathrm{IW}(\boldsymbol{X}_k; v_{k|k-1}, \boldsymbol{V}_{k|k-1})\pi_k^{\gamma_k}(1-\pi_k)^{(1-\gamma_k)}\\
&\cdot [N(\boldsymbol{z}_k^j; \tilde{\boldsymbol{H}}_k\boldsymbol{x}_k, (s\boldsymbol{X}_k + \boldsymbol{R}_k)/n_k]^{\gamma_k}\\
&\cdot \mathrm{Be}(\pi_k; g_0, 1-g_0)\\
&\cdot [N(\boldsymbol{z}_k; \tilde{\boldsymbol{H}}_k\boldsymbol{x}_k, (s\boldsymbol{X}_k + \boldsymbol{R}_k)/n_k\rho_k)]^{(1-\gamma_k)}\\
&\cdot \left[\mathrm{LW}(\bar{\boldsymbol{Z}}^k; n_k - 1, s\boldsymbol{X}_k + \boldsymbol{R}_k)\right]^{\gamma_k}\\
&\cdot \mathrm{Be}(\tau_k; q_0, 1-q_0)\\
&\cdot \left[\mathrm{LW}(\bar{\boldsymbol{Z}}^k; n_k - 1, (s\boldsymbol{X}_k + \boldsymbol{R}_k)/\rho_k)\right]^{(1-\gamma_k)}\\
&\cdot G\left(\xi_k; \frac{\omega_k}{2}, \frac{\omega_k}{2}\right) G\left(\lambda_k; \frac{u_k}{2}, \frac{u_k}{2}\right)\\
&\cdot \tau_k^{t_k}(1-\tau_k)^{(1-t_k)}
\end{aligned} \tag{26}$$

(1)当$\varphi = \boldsymbol{x}_k$时，可将$q^{(i+1)}(\boldsymbol{x}_k)$更新为高斯分布

$$q^{(i+1)}(\boldsymbol{x}_k) = N(\boldsymbol{x}_k; \boldsymbol{x}_{k|k}^{(i+1)}, \boldsymbol{P}_{k|k}^{(i+1)}) \tag{27}$$

其中

$$\boldsymbol{K}_{k|k-1}^{(i+1)} = \tilde{\boldsymbol{P}}_{K|k-1}^{(i)}\tilde{\boldsymbol{H}}_k^{\mathrm{T}}\left(\boldsymbol{S}_{k|k-1}^{(i+1)}\right)^{-1} \tag{28}$$

$$\boldsymbol{x}_{k|k}^{(i+1)} = \boldsymbol{x}_{k|k-1} + \left(\boldsymbol{K}_{k|k-1}^{(i+1)}\otimes\boldsymbol{I}_d\right)(\boldsymbol{z}_k - \tilde{\boldsymbol{H}}_k\boldsymbol{x}_{k|k-1}) \tag{29}$$

$$\boldsymbol{P}_{k|k}^{(i+1)} = \tilde{\boldsymbol{P}}_{K|k-1}^{(i)} - \boldsymbol{K}_{k|k-1}^{(i+1)}\boldsymbol{S}_{k|k-1}^{(i+1)}\left(\boldsymbol{K}_{k|k-1}^{(i+1)}\right)^{\mathrm{T}} \tag{30}$$

$$\boldsymbol{S}_{k|k-1}^{(i+1)} = \tilde{\boldsymbol{H}}_k\tilde{\boldsymbol{P}}_{K|k-1}^{(i)}\tilde{\boldsymbol{H}}_k^{\mathrm{T}} + \frac{s\boldsymbol{X}_k^{(i)} + \tilde{\boldsymbol{R}}_k^{(i)}}{n_k} \tag{31}$$

其中，参数$\boldsymbol{K}_{k|k-1}^{i+1}$和$\boldsymbol{S}_{k|k-1}^{i+1}$表示增益矩阵和标量创新因子，其中$\tilde{\boldsymbol{P}}_{k|k-1}^{(i)}$和$\tilde{\boldsymbol{R}}_k^{(i)}$可表示为

$$\tilde{\boldsymbol{P}}_{K|k-1}^{(i)} = \frac{\boldsymbol{P}_{k|k-1}}{E^{(i)}[t_k] + (1-E^{(i)}[t_k])E^{(i)}[\sigma_k]} \tag{32}$$

$$\tilde{\boldsymbol{R}}_k^{(i)} = \frac{\boldsymbol{R}_k}{E^{(i)}[\gamma_k] + (1-E^{(i)}[\gamma_k])E^{(i)}[\rho_k]} \tag{33}$$

(2)当$\varphi = \boldsymbol{X}_k$时，可将$q^{(i+1)}(\boldsymbol{X}_k)$更新为逆威沙特分布

$$q^{(i+1)}(\boldsymbol{X}_k) = \mathrm{IW}\left(\boldsymbol{X}_k^{(i+1)}; v_{k|k}^{(i+1)}, \boldsymbol{V}_{k|k}^{(i+1)}\right) \tag{34}$$

其中

$$v_{k|k} = v_{k|k-1} + n_k \tag{35}$$

$$E^{(i+1)}[\boldsymbol{X}_k] = \frac{\boldsymbol{V}_{k|k}^{(i+1)}}{v_{k|k} - 2d - 2} \tag{36}$$

$$\boldsymbol{V}_{k|k}^{(i+1)} = \boldsymbol{V}_{k|k-1} + \left(\boldsymbol{S}_{k|k-1}^{(i+1)}\right)^{-1}\boldsymbol{N}_{k|k-1} + \bar{\boldsymbol{Z}}_k \tag{37}$$

$$\boldsymbol{N}_{k|k-1} = (\boldsymbol{z}_k - \tilde{\boldsymbol{H}}_k\boldsymbol{x}_{k|k-1})(\boldsymbol{z}_k - \tilde{\boldsymbol{H}}_k\boldsymbol{x}_{k|k-1})^{\mathrm{T}} \tag{38}$$

其中，$\boldsymbol{N}_{k|k-1}$为量测误差协方差矩阵。

(3)当$\varphi = t_k$和$\varphi = \gamma_k$时，可将$q^{(i+1)}(t_k)$和$q^{(i+1)}.(\gamma_k)$更新为伯努利分布，当t_k和γ_k分别取0或1时的概率为

$$p_{t_k=1}^{(i+1)} = \exp\left\{E^{(i)}[\ln\tau_k] - 0.5\mathrm{tr}\left(\boldsymbol{A}_k^{(i+1)}\boldsymbol{P}_{k|k-1}^{-1}\right)\right\} \tag{39}$$

$$p_{\gamma_k=1}^{(i+1)} = \exp\left\{E^{(i)}[\ln\pi_k] - 0.5\mathrm{tr}\left(\boldsymbol{B}_k^{(i+1)}\boldsymbol{R}_k^{-1}\right)\right\} \tag{40}$$

$$\begin{aligned}
p_{t_k=0}^{i+1} = \exp\Big\{&E^{(i)}[\ln(1-\tau_k)] + 0.5nE^{(i)}[\ln\sigma_k]\\
&-0.5E^{(i)}[\sigma_k]\mathrm{tr}\left(\boldsymbol{A}_k^{i+1}\boldsymbol{P}_{k|k-1}^{-1}\right)\Big\}
\end{aligned} \tag{41}$$

$$\begin{aligned}
p_{\gamma_k=0}^{i+1} = \exp\Big\{&E^{(i)}[\ln(1-\pi_k)] + 0.5nE^{(i)}[\ln\rho_k]\\
&-0.5E^{(i)}[\rho]\mathrm{tr}\left(\boldsymbol{B}_k^{i+1}\boldsymbol{R}_k^{-1}\right)\Big\}
\end{aligned} \tag{42}$$

其中，n和m分别代表运动状态和量测的维数；辅助矩阵$\boldsymbol{A}_k^{(i+1)}$, $\boldsymbol{B}_k^{(i+1)}$可表示为

$$\begin{aligned}
\boldsymbol{A}_k^{(i+1)} = &\boldsymbol{P}_{k|k}^{(i+1)} + \left(\boldsymbol{x}_{k|k}^{(i+1)} - \boldsymbol{x}_{k|k-1}\right)\\
&\cdot\left(\boldsymbol{x}_{k|k}^{(i+1)} - \boldsymbol{x}_{k|k-1}\right)^{\mathrm{T}}
\end{aligned} \tag{43}$$

$$\begin{aligned}
\boldsymbol{B}_k^{(i+1)} = &\tilde{\boldsymbol{H}}_k\boldsymbol{P}_{k|k}^{(i+1)}\tilde{\boldsymbol{H}}_k^{\mathrm{T}} + \left(\boldsymbol{z}_k - \tilde{\boldsymbol{H}}_k\boldsymbol{x}_{k|k}^{(i+1)}\right)\\
&\cdot\left(\boldsymbol{z}_k - \tilde{\boldsymbol{H}}_k\boldsymbol{x}_{k|k}^{(i+1)}\right)^{\mathrm{T}}
\end{aligned} \tag{44}$$

⑷当$\varphi=\sigma_k$和$\varphi=\rho_k$时，可将$q^{i+1}(\sigma_k)$和$q^{i+1}(\rho_k)$更新为伽马分布

$$q^{(i+1)}(\sigma_k)=G\left(\xi_k;\eta_k^{(i+1)},\vartheta_k^{(i+1)}\right) \tag{45}$$

$$q^{(i+1)}(\rho_k)=G\left(\lambda_k;\alpha_k^{(i+1)},\beta_k^{(i+1)}\right) \tag{46}$$

其中，形状参数$\eta_k^{(i+1)}$, $\alpha_k^{(i+1)}$和速率参数$\vartheta_k^{(i+1)}$, $\beta_k^{(i+1)}$可表示为

$$\eta_k^{(i+1)}=0.5n\left(1-E^{(i+1)}[t_k]\right)+0.5\omega_k \tag{47}$$

$$\alpha_k^{(i+1)}=0.5m\left(1-E^{(i+1)}[\gamma_k]\right)+0.5u_k \tag{48}$$

$$\vartheta_k^{(i+1)}=0.5\mathrm{tr}\left(\boldsymbol{A}_k^{(i+1)}\boldsymbol{P}_{k|k-1}^{-1}\right)\left(1-E^{(i+1)}[t_k]\right)+0.5\omega_k \tag{49}$$

$$\beta_k^{(i+1)}=0.5\mathrm{tr}\left(\boldsymbol{B}_k^{(i+1)}\boldsymbol{R}_k^{-1}\right)\left(1-E^{(i+1)}[\gamma_k]\right)+0.5u_k \tag{50}$$

其中，γ_k和t_k的期望可表示为

$$\begin{aligned}E^{(i+1)}[t_k]&=\frac{p_{t_k=1}^{(i+1)}}{p_{t_k=1}^{(i+1)}+p_{t_k=0}^{(i+1)}}\\E^{(i+1)}[\gamma_k]&=\frac{p_{\gamma_k=1}^{(i+1)}}{p_{\gamma_k=1}^{(i+1)}+p_{\gamma_k=0}^{(i+1)}}\end{aligned} \tag{51}$$

⑸当$\varphi=\tau_k$和$\varphi=\pi_k$时，可将$q^{i+1}(\tau_k)$和$q^{i+1}.(\pi_k)$更新为贝塔分布

$$q^{(i+1)}(\tau_k)=\mathrm{Be}\left(\tau_k;g_k^{(i+1)},h_k^{(i+1)}\right) \tag{52}$$

$$q^{(i+1)}(\pi_k)=\mathrm{Be}\left(\pi_k;e_k^{(i+1)},f_k^{(i+1)}\right) \tag{53}$$

其中，形状参数$g_k^{(i+1)}$, $h_k^{(i+1)}$, $e_k^{(i+1)}$和$f_k^{(i+1)}$可表示为

$$g_k^{(i+1)}=q_0+E^{(i+1)}[t_k],h_k^{(i+1)}=2-q_0-E^{(i+1)}[t_k] \tag{54}$$

$$e_k^{(i+1)}=g_0+E^{(i+1)}[\gamma_k],f_k^{(i+1)}=2-g_0-E^{(i+1)}[\gamma_k] \tag{55}$$

(6)根据式(52)和式(53)，$E^{(i+1)}[\ln\tau_k]$, $E^{(i+1)}[\ln\pi_k]$, $E^{(i+1)}[\ln(1-\tau_k)]$和$E^{(i+1)}[\ln(1-\pi_k)]$可表示为

$$E^{(i+1)}[\ln\tau_k]=\psi\left(g_k^{(i+1)}\right)-\psi\left(g_k^{(i+1)}+h_k^{(i+1)}\right) \tag{56}$$

$$E^{(i+1)}[\ln\pi_k]=\psi\left(e_k^{(i+1)}\right)-\psi\left(e_k^{(i+1)}+f_k^{(i+1)}\right) \tag{57}$$

$$E^{(i+1)}[\ln(1-\tau_k)]=\psi\left(h_k^{(i+1)}\right)-\psi\left(g_k^{(i+1)}+h_k^{(i+1)}\right) \tag{58}$$

$$E^{(i+1)}[\ln(1-\pi_k)]=\psi\left(f_k^{(i+1)}\right)-\psi\left(e_k^{(i+1)}+f_k^{(i+1)}\right) \tag{59}$$

(7)根据式(45)和式(46)，$E^{(i+1)}[\sigma_k]$, $E^{(i+1)}[\rho_k]$, $E^{(i+1)}[\ln\sigma_k]$和$E^{(i+1)}[\ln\rho_k]$可表示为

$$E^{(i+1)}[\sigma_k]=\frac{\eta_k^{(i+1)}}{\vartheta_k^{(i+1)}},E^{(i+1)}[\rho_k]=\frac{\alpha_k^{(i+1)}}{\beta_k^{(i+1)}} \tag{60}$$

$$E^{(i+1)}[\ln\sigma_k]=\psi\left(\eta_k^{(i+1)}\right)-\ln\vartheta_k^{(i+1)} \tag{61}$$

$$E^{(i+1)}[\ln\rho_k]=\psi\left(\alpha_k^{(i+1)}\right)-\ln\beta_k^{(i+1)} \tag{62}$$

4　仿真实验

4.1　设置

为验证本文所提算法的有效性，在非平稳异常噪声环境下对比Student's t逆Wishart(Student's t Inverse Wishart, StIW)算法[27]、正态-伽马分布下的扩展目标(Normal Gamma, NG)算法、最大相关熵随机矩阵 (Maximum Correntropy Random Matrix Model, MCRMM) 算法和本文的GSTM随机矩阵 (Gaussian-Student's T Mixture Random Matrix Model, GSTMRMM) 算法的跟踪效果。设定量测维数$d=2$，采样周期$T=1$ s，蒙特卡罗仿真次数为100，总采样次数为100。设过程噪声$\boldsymbol{D}_{k|k-1}=0.1^2$，目标的时间演化方程和量测方程如式(2)和式(3)所示，其中$\boldsymbol{F}_{k|k-1}=\begin{bmatrix}1&T\\0&1\end{bmatrix}$, $\boldsymbol{H}_k=[1\ 0]$。

目标的初始化参数为

$$v_0=7,\ \boldsymbol{V}_0=\begin{pmatrix}1&0\\0&1\end{pmatrix},\ \boldsymbol{x}_0=[30,10,15,15]^{\mathrm{T}} \tag{63}$$

异常噪声可设置为

$$w_k\sim\begin{cases}N(0,\boldsymbol{Q}_k),\text{w.p.}0.75,1\le k\le 20,50<k\le 80\\N(0,1000\boldsymbol{Q}_k),\text{w.p.}0.25,1\le k\le 20,50<k\le 80\\N(0,\boldsymbol{Q}_k),\text{w.p.}1,20<k\le 50,50<k\le 100\end{cases} \tag{64}$$

$$v_k\sim\begin{cases}N(0,\boldsymbol{R}_k),\text{w.p.}0.85,1\le k\le 20,50<k\le 80\\N(0,100\boldsymbol{R}_k),\text{w.p.}0.15,1\le k\le 20,50<k\le 80\\N(0,\boldsymbol{R}_k),\text{w.p.}1,20<k\le 50,80<k\le 100\end{cases} \tag{65}$$

式(64)和式(65)通过设置不同概率和时间段，仿真实验中所遇到的非平稳异常噪声。在实验过程中需要用VB方法，将最大迭代次数和收敛准则分别设定为$M=50$和$e=10^{-16}$[22]。

4.2　实验仿真1

本次实验设定椭圆的长短轴在跟踪过程中保持

不变，分别为$a=8$和$b=4$。图1为不同算法的匀速直线运动轨迹跟踪图。从图1中可以观察到，由于跟踪过程中存在非平稳的厚尾噪声，会使扩展目标在运动过程中产生机动。而4种算法也都能对扩展目标轨迹进行有效跟踪。为更全面分析不同滤波器在非平稳异常噪声下的跟踪性能，需对比4种算法滤波后质心估计和形状估计的准确性。

为验证算法在目标质心跟踪中的有效性，图2为100次蒙特卡罗实验下4种算法的目标质心跟踪误差统计平均图。由图2可清晰看出，4种算法对目标质心的跟踪误差都较小。NG算法、StIW算法和MCRMM算法对非高斯噪声都进行了建模，在厚尾噪声条件下有着良好的估计精度，但是当噪声切换为高斯噪声时，精度有所下降。而本文所提GSTMRMM算法考虑到了高斯-厚尾噪声之间的非平稳随机变换，不仅能以极快的速度进行收敛，还能自如地适应不同噪声之间的切换，在跟踪过程中展现出来较强的稳定性。

在扩展目标跟踪中，算法对形状的自适应估计是评判跟踪效果的重要指标。图3给出了3个典型阶段的目标形状估计放大图，由图3(a)可以看出，由于目标的先验信息未知，4种算法在初始阶段对目标形状的估计都比较粗糙。随着滤波不断更新，4种算法对轮廓的估计精度不断提升。而GSTMRMM算法对非平稳异常噪声有着更为精确的建模，相较于其他3种滤波器，算法呈现出对非平稳异常噪声更好的抗干扰性。

图4给出了扩展目标跟踪的高斯-瓦瑟斯坦距离，通过GWD可以综合评估目标运动质心跟踪效果和扩展状态跟踪效果。仔细观察图4可得，GSTMRMM在质心跟踪和形状跟踪上均表现出快速的收敛性能。而GWD图进一步佐证了GSTMRMM算法在复杂噪声中对扩展目标跟踪的优越性与稳定性。

4.3 实验仿真2

为进一步验证所提算法的鲁棒性与适应性，在上述实验基础上，设定椭圆长短轴和角度存在动态演变，变化率分别为$r_l=1.01$和$r_\theta=1.05$。图5为形状演变下的轨迹跟踪图，从图5中可以清晰观察到，GSTMRMM算法仍能实现对目标位置的准确预测，表明GSTMRMM在形状演变的过程中具有较强的鲁棒性。

图6展示了100次蒙特卡罗仿真下质心估计的

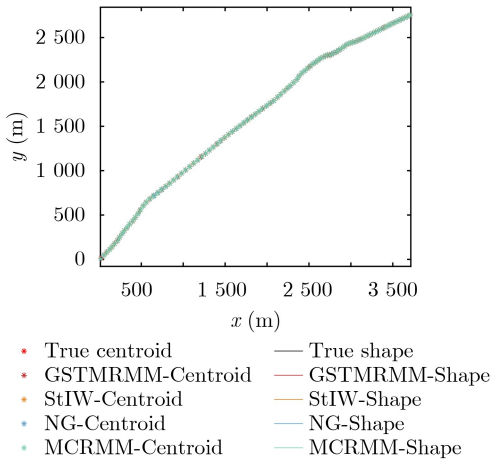


图 1 扩展目标轨迹跟踪图

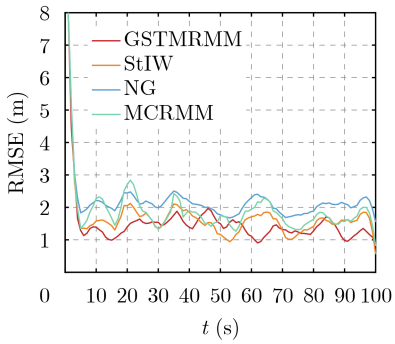
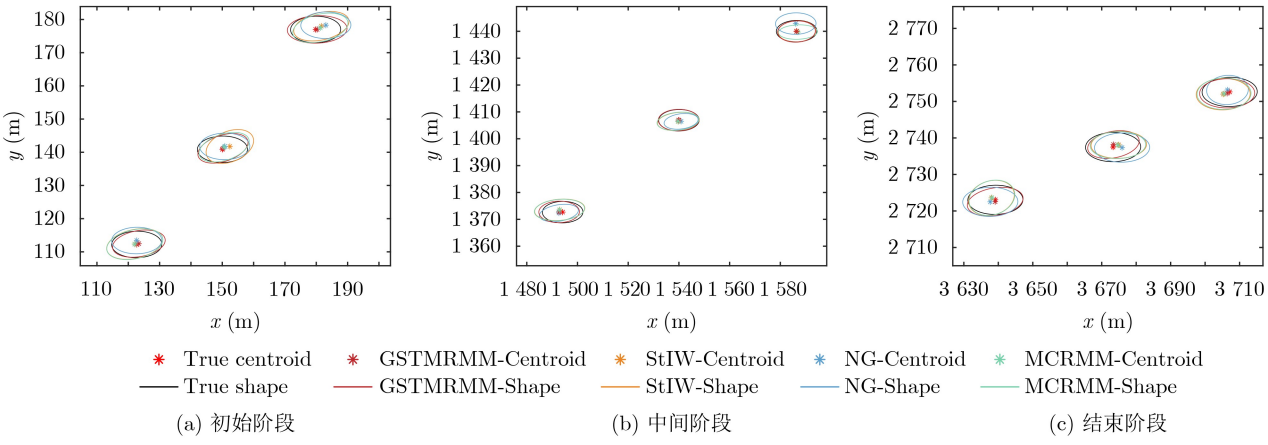


图 2 目标质心跟踪误差



(a) 初始阶段 (b) 中间阶段 (c) 结束阶段

图 3 不同阶段的轮廓放大图

RMSE，从图中可以看出，GSTMRMM算法的跟踪效果明显优于其他3种算法。StIW，NG和MCRMM虽然能在很大程度上包容厚尾噪声，然而受非平稳噪声的影响，估计精度会有所下降。相比之下，GSTMRMM算法在目标定位跟踪过程中依然表现出较强的稳定性，能够更有效地处理噪声的非平稳性。

图7将目标运动过程3个代表性阶段的轮廓放大图展现出。结果清晰表明，尽管目标的长短轴和方向角在不断变化，为目标跟踪过程中增加了更多的不确定因素，但GSTMRMM 算法仍能精确捕捉这些变化，跟踪精度明显优于其他算法。并通过图8的GWD结果更客观地反映算法对扩展目标的跟踪精度，验证了GSTMRMM 算法在形状动态演变过程中仍能稳健地估计目标的运动状态，并精准捕捉目标轮廓。

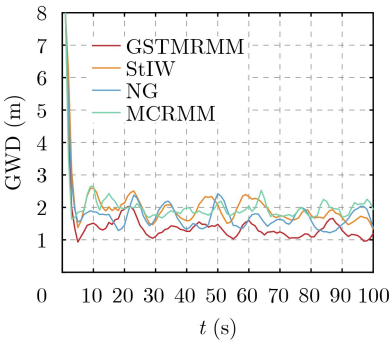


图 4 椭圆扩展目标GWD

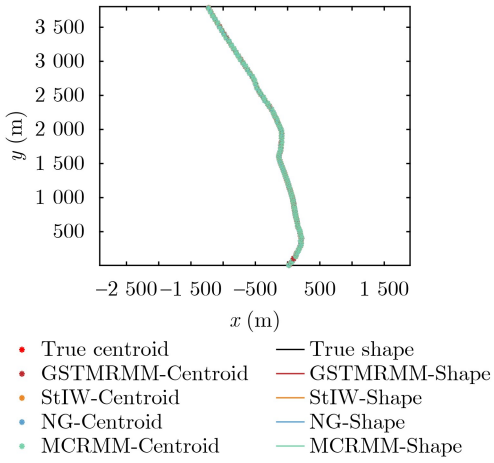


图 5 扩展目标轨迹跟踪图

4.4 实验仿真3(真实场景测试)

本节对4种滤波算法在真实场景下的跟踪效果进行了对比分析。该场景为航空摄像机每秒捕获的车辆图像，即$T=1$ s，共采集了105个时刻的数据。通过图像特征提取及车辆像素点均匀采样，获取测量信息。在目标车辆运动过程中，由于树枝树叶遮挡，量测值发生异常，使得过程噪声和量测噪声呈现“厚尾特性”。当车辆行驶到空旷地区时，环境中的干扰因素显著减少，噪声转变为高斯噪声。因此，在车辆运动过程中，噪声表现出非平稳特性。通过此实验场景，可以有效验证所提出算法在非平稳异常噪声环境下的稳健性。

真实场景中，目标的初始位置由第1帧图像获取，并假设车辆的初始速度为未知的，设定为0，即目标的初始均值为$\boldsymbol{x}_0=[450,245,0,0]^{\mathrm{T}}$。将目标

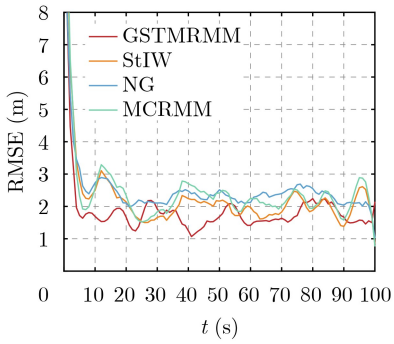
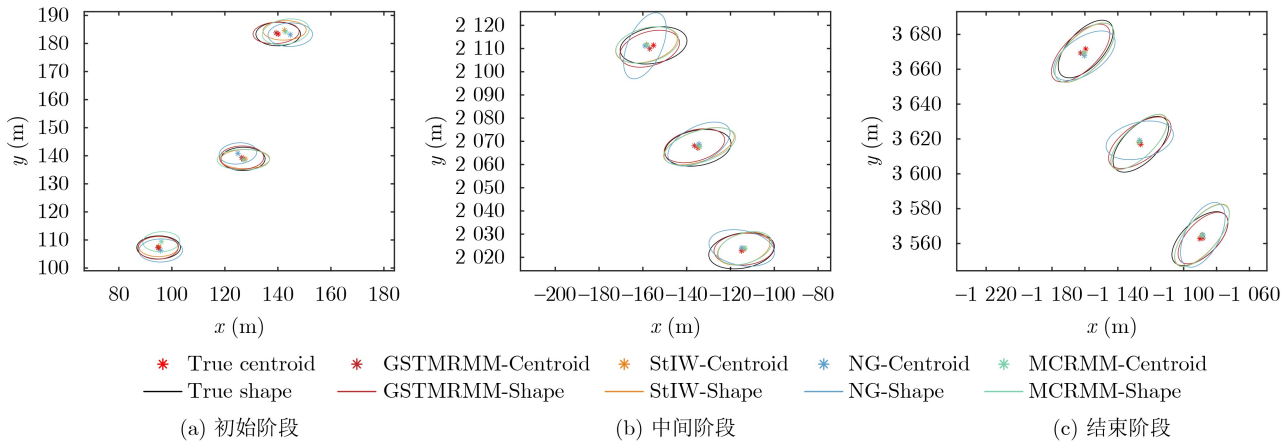


图 6 目标质心跟踪误差



(a) 初始阶段　　(b) 中间阶段　　(c) 结束阶段

图 7 不同阶段的轮廓放大图

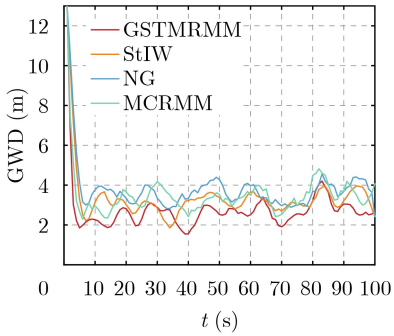


图 8 椭圆扩展目标GWD

扩展状态的初始自由度和尺度矩阵分别设为$v_0=7$和$\boldsymbol{V}_0=([250,1\,000])$。为清晰对比算法性能差异，图9展现了$\{3,19,29,39,86,94\}$时刻4种算法估计车辆运动状态和扩展状态的效果图。

仔细观察图9可知，在图9(a)中，车辆保持静止时，4种算法能较好地估计出目标轮廓；当车辆匀速直线运动时，如图9(b)所示，4种算法都能对目标轮廓有效跟踪，但本文算法对轮廓的估计更为精准；在图9(c)中，目标车辆做转弯运动，相较于其他3种算法，GSTMRMM算法对方向的估计精度更高，但由于量测数据集中在车辆前半部分，未能完全捕捉轮廓；当车辆经过树枝遮挡路段时，如图9(e)所示，噪声为厚尾噪声，GSTMRMM和StIW呈现出更优越的跟踪效果；在图9(f)中，车辆驶出复杂路段，噪声切换为高斯噪声，GSTMRMM能自适应更新噪声模型实现精准跟踪。该实验场景下，车辆运动过程中存在机动或遮挡等复杂因素，综合所有路况下的实验效果，由于GSTMRMM算法能对非平稳异常噪声精确建模，显著提升了车辆位置和形状的跟踪精度，这与实验仿真中的结果相吻合。

图 9 不同时刻效果图

5 结束语

本文针对非平稳异常噪声条件下扩展目标跟踪问题，提出了GSTM扩展目标跟踪方法。采用GSTM分布对非平稳噪声建模，并利用VB方法推导出复杂环境下的扩展目标跟踪算法。通过多重实验表明，本算法能够在非平稳异常噪声环境中能提供可靠且高精度的目标跟踪性能。在今后的研究中，可以通过多目标跟踪理论将算法扩展到多扩展目标跟踪中，为复杂环境下的多扩展目标跟踪提供更加可靠的解决方案。

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Extended Target Tracking Method under Non-stationary Abnormal Noise Conditions

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Abstract:

Objective This paper addresses the problem of extended target tracking in the presence of non-stationary abnormal noise. Traditional Gaussian extended target filters and Student's t filters rely on the assumption of stationary noise distributions, which limits their performance in environments with non-stationary abnormal noise. Non-stationary noise, common in practical applications, is especially prevalent in complex environments where the noise frequently shifts between Gaussian and heavy-tailed distributions. To overcome this challenge, a Gaussian-Student's t Mixture (GSTM) distribution is proposed for modeling non-stationary abnormal noise in extended target tracking. The GSTM distribution is used to model the noise accurately, and a filter is developed to track the target's kinematic state and shape effectively under non-stationary measurement and process noise conditions. This method is shown to be robust in complex environments, offering enhanced accuracy, robustness, and applicability for extended target tracking.

Methods The GSTM distribution is employed to model both process and measurement noise, enabling dynamic adjustment of mixture parameters to capture the evolving characteristics of noise distributions in non-stationary environments. To optimize computation, Bernoulli random variables are introduced, and the target's one-step prediction and measurement likelihood functions are reformulated as a hierarchical Gaussian model based on the GSTM distribution. This approach facilitates adaptive switching between Gaussian and Student's t distributions, streamlining the inference process and simplifying posterior computation, which reduces the complexity of parameter estimation. Within the Random Matrix Model (RMM) framework, Variational Bayesian (VB) inference is applied to jointly estimate the target's kinematic state, extension state, mixture parameters, and noise characteristics. During the filtering update phase, a dynamic adjustment mechanism is introduced for the one-step prediction error covariance matrix and observation noise covariance matrix, ensuring the model to maintain robustness and adaptability in complex, non-stationary noise environments.

Results and Discussions The introduction of the GSTM distribution for modeling non-stationary abnormal noise enables robust tracking of both the centroid and shape contour of extended targets in such environments. Theoretical derivations and experimental validations confirm the effectiveness of the proposed method for single extended target tracking under non-stationary noise conditions. Simulation and real-world results demonstrate significant performance advantages. First, in terms of tracking accuracy, the proposed algorithm achieves a

notably lower Root Mean Square Error (RMSE) for centroid tracking compared to other algorithms (Fig. 2, Fig. 6), effectively adapting to dynamic changes in non-stationary noise, and offering superior accuracy and stability. Second, for adaptive estimation of target shape, the algorithm shows considerable improvements in non-stationary noise environments, providing more accurate contour estimation (Fig. 3, Fig. 7). It also maintains high robustness under evolving target shapes. Moreover, the algorithm exhibits faster convergence and greater stability in complex environments (Fig. 2, Fig. 4), with a significantly lower Gaussian Wasserstein Distance (GWD) mean compared to other methods (Fig. 4, Fig. 8). In practical experiments, a vehicle operated in environments with obstacles like tree branches, where the noise is non-stationary, further validated the algorithm's performance. Under these conditions, the proposed algorithm demonstrated exceptional stability and robustness throughout the tracking process (Fig. 9), outperforming other algorithms and highlighting its adaptability and reliability in complex dynamic environments.

Conclusions　This paper proposes an extended target tracking method based on the GSTM distribution, overcoming the limitations of traditional algorithms in adapting to non-stationary anomalous noise environments. The GSTM distribution is used for noise modeling, combined with the RMM framework, and the VB method along with hierarchical Gaussian modeling simplifies the computational process, enhancing the algorithm's adaptability and robustness. Experimental results across shape-invariant, shape-evolving, and real-world scenarios demonstrate the following: (1) The proposed algorithm significantly outperforms existing methods in robustness, particularly in centroid tracking and shape estimation. (2) The noise model is adaptively adjusted under non-stationary noise and dynamic target evolution, enabling high-precision tracking of extended targets. (3) In complex real-world scenarios, the algorithm successfully tracks small vehicles, further validating its effectiveness in practical applications. Future research could explore integrating multi-target tracking theories, extending the algorithm to multi-extended target tracking scenarios, and addressing more complex environmental challenges to further enhance its practicality and performance in multi-target settings.

Key words: Extended target tracking; Random matrix; Gaussian-Student's t Mixture (GSTM) distribution; Variational Bayesian (VB) method