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静电迴旋单腔管起振电流和频率偏移的研究* **

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摘要　本文从线性 Vlasov-Maxwell 方程出发，求得了具有纵向场分布 $\sin k_{\parallel}z$ 的静电单腔管的起振电流和频偏表达式，并进行了数值计算.

关键词　静电电子回旋脉塞；静电迴旋单腔管；起振电流；频率偏移

1. 引言

1986年刘盛纲等[1]提出了静电电子回旋脉塞的概念，并发现了以 $\Omega = \omega - k_{\parallel}V_{\parallel} - m\omega_0 = 0$ 和以 $\Omega_r^2 = (2-\beta_{\perp}^2)\omega_0^2 - \Omega^2 = 0$ 为基础的不稳性. 本文报道了采用动力学理论建立起的静电单腔管线性理论，对静电迴旋单腔管的起振特性进行了研究，求得了起振电流和频率偏移的表达式，并进行了数值计算.

2. 静电单腔管的动力学描述

相对论性线性 Vlasov 方程为

$$\frac{\partial f_1}{\partial t} + \boldsymbol{V}\cdot\nabla_r f_1 + e(\boldsymbol{E}_1 + \boldsymbol{V}\times\boldsymbol{B}_1)\cdot\nabla_p f_0 + e(\boldsymbol{E}_0)\cdot\nabla_p f_0 = 0 \tag{1}$$

式中 f_1 为一阶扰动分布函数，$\boldsymbol{E}_1$ 和 $\boldsymbol{B}_1$ 分别为高频电场和磁场，$\boldsymbol{E}_0$ 为静电场，$\boldsymbol{V}$ 为电子运动速度，而 f_0 为平衡分布函数. 在作圆轨道的一级近似条件下，平衡分布函数为[2]

$$f_0 = \frac{\sqrt{2-\beta_{\varphi 0}}}{2\pi^2} n_l \frac{P_0}{P_{\varphi 0}} \delta(H-H_0)\delta(P_\varphi - P_{\varphi 0})\delta(p_z - p_{z0}) \tag{2}$$

式中 $\beta_{\varphi 0} = V_{\varphi 0}/c$，$V_{\varphi 0}$ 和 c 分别为电子角向初速度和真空中的光速；n_l 表示电子线密度；H、P_φ 和 p_z 分别为总能量、正则角动量和力学动量；而 H_0、$P_{\varphi 0}$ 和 p_{z0} 则分别为其初始值.

在极坐标中，同轴腔中 TE_{mn} 模的高频场表达式为：

$$B_R = \mp j\mu_0 k_{\parallel} k_c Z_m'(k_c R) \tag{3a}$$

$$B_\varphi = \pm \mu_0 k_{\parallel}\left(\frac{m}{R}\right) Z_m(k_c R) \tag{3b}$$

$$B_z = \mu_0 k_c^2 Z_m(k_c R) \tag{3c}$$

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$$E_R = \frac{\omega}{k_{/\!/}} B_\varphi \tag{3d}$$

$$E_\varphi = -\frac{\omega}{k_{/\!/}} B_R \tag{3e}$$

$$E_z = 0 \tag{3f}$$

式中略去了正向波相位因子 $e^{j\theta}$, $\theta = -j(\omega t - k_{/\!/} z - m\varphi)$; 返向波相位因子 $e^{j\theta'}$, $\theta' = -j(\omega t + k_{/\!/} z - m\varphi)$; 以及腔体端面反射系数 Γ。

采用沿未扰轨道积分的方法，一阶扰动分布函数为

$$f_1 = |e| \int_{t-\frac{z}{V_z}}^{t} (\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B}) \cdot \nabla_p f_0 dt' \tag{4}$$

将(2)式和 (3a)—(3f) 式代入(4)式，最后得到正向波和返向波的一阶扰动分布函数分别为

$$f_1 = -|e| \mu_0 k_{/\!/} k_c Z'_m(k_c R) \left[V_\varphi \left(V_p \frac{\partial f_0}{\partial H} + \frac{\partial f_0}{\partial p_z} \right) + \Delta V R \frac{\partial f_0}{\partial P_\varphi} \right] \times \left(\frac{1 - e^{j\Omega z/V_z}}{\Omega} \right) e^{j(-\omega t + k_{/\!/} z + m\varphi)}. \tag{5}$$

$$f'_1 = \Gamma |e| \mu_0 k_{/\!/} k_c Z'_m(k_c R) \left[V_\varphi \left(V_p \frac{\partial f_0}{\partial H} + \frac{\partial f_0}{\partial p_z} \right) + \Delta V' \frac{\partial f_0}{\partial P_\varphi} \right] \times \left(\frac{1 - e^{j\Omega' z/V_z}}{\Omega'} \right) e^{j(-\omega t - k_{/\!/} z + m\varphi)}. \tag{6}$$

式中 $\Delta V = V_p - V_z$, $\Delta V' = V_p + V_z$, $\Omega = \omega - k_{/\!/} V_z - m\omega_c$, $\Omega' = \omega + k_{/\!/} V_z - m\omega_c$, $V_p = \omega / k_{/\!/}$。

电子注与波互作用的全功率为

$$P_e = \frac{1}{2} \int_v (\boldsymbol{J}_\varphi \cdot \boldsymbol{E}^*_\varphi) dv \tag{7}$$

以及

$$\boldsymbol{J} = e \int f_1 \boldsymbol{V} d^3 p \tag{8}$$

并由起振功率条件，最后得起振电流为

$$I_{st} = -\frac{\pi (2 - \beta^2_{\varphi 0}) k_c^2 R_b^2 V_{z0} \gamma_0 \left(1 - \frac{R_a^2}{R_b^2}\right) H_m}{\eta_0 \mu_0 \left(1 - \frac{q^2 \lambda_0^2}{4L^2}\right) Q_T \left[\sum_{i=1}^{4} \overline{\mathrm{Re}(P_{ei})} \right]} \tag{9}$$

频偏为

$$\frac{\Delta\omega}{\omega} = \frac{\nu \left[\sum_{i=1}^{4} \overline{\mathrm{Im}(P_{ei})} \right] \left(1 - \frac{q^2 \lambda_0^2}{4L^2}\right)}{\gamma_0 (2 - \beta^2_{\varphi 0}) k_c^2 R_b^2 \pi \left(1 - \frac{R_a^2}{R_b^2}\right) H_m} \tag{10}$$

式中 η_0 为荷质比; $L = q\lambda_g/2$, q 表示波在腔体内的半波长数, λ_g 为波导波长;

$$\overline{\mathrm{Re}(P_{ei})} = \mathrm{Re}(P_{ei}) \Big/ \left(\nu k_c^2 \mu_0 \frac{\omega}{\gamma_0} \frac{L}{2 - \beta^2_{\varphi 0}} \right)$$

$$\overline{\mathrm{Im}(P_{ei})} = \mathrm{Im}(P_{ei}) \Big/ \left(\nu k_c^2 \mu_0 \frac{\omega}{\gamma_0} \frac{L}{2-\beta_{\varphi 0}^2}\right)$$

$$H_m = J_m(k_c R) N'_m(k_c R_a) - N_m(k_c R) J'_m(k_c R_a)$$

而

$$\mathrm{Re}(P_{e1}) = \nu k_c^2 \mu_0 \frac{\omega}{\gamma_0} \frac{L^2}{2-\beta_{\varphi 0}^2} \left\{\left(\frac{A_1}{\phi^2 V_{z0}} + \frac{B_1 L}{\phi^3 V_{z0}^2} + \frac{D_1 L}{\phi^3 V_{z0}}\right)(\cos\phi - 1) + \frac{D_1 L}{\phi^2 V_{z0}} \sin\phi\right\} \tag{11a}$$

$$\mathrm{Im}(P_{e1}) = \nu k_c^2 \mu_0 \frac{\omega}{\gamma_0} \frac{L}{2-\beta_{\varphi 0}^2} \left\{\left(\frac{A_1 L}{\phi V_{z0}} + \frac{B_1 L^2}{\phi^2 V_{z0}^2}\right)\left(\frac{1}{\phi}\sin\phi - 1\right) + \frac{D_1 L}{\phi V_{z0}} \times \left(\frac{L}{\phi^2}\sin\phi - \frac{L}{\phi}\cos\phi\right)\right\} \tag{11b}$$

$$\mathrm{Re}(P_{e2}) = \Gamma^2 \nu \mu_0 k_c^2 \frac{\omega}{\gamma_0} \frac{L^2}{2-\beta_{\varphi 0}^2} \left\{\left(\frac{A_2}{\phi'^2 V_{z0}} + \frac{B_2 L}{\phi'^3 V_{z0}^2} + \frac{D_2 L}{\phi'^3 V_{z0}}\right)(\cos\phi' - 1) + \frac{D_2 L}{\phi'^2 V_{z0}} \sin\phi'\right\} \tag{11c}$$

$$\mathrm{Im}(P_{e2}) = \Gamma \nu k_c^2 \mu_0 \frac{\omega}{\gamma_0} \frac{L}{2-\beta_{\varphi 0}^2} \left\{\left(\frac{A_2 L}{\phi' V_{z0}} + \frac{B_2 L^2}{\phi'^2 V_{z0}}\right)\left(\frac{1}{\phi'}\sin\phi' - 1\right) + \frac{D_2 L}{\phi' V_{z0}}\left(\frac{L}{\phi'^2}\sin\phi' - \frac{L}{\phi'}\cos\phi'\right)\right\} \tag{11d}$$

$$\mathrm{Re}(P_{e3}) = \Gamma \nu k_c^2 \mu_0 \frac{\omega}{\gamma_0} \frac{L}{2-\beta_{\varphi 0}^2} \left[\left(\frac{A_3 L}{\phi V_{z0}} + \frac{B_3 L^2}{\phi^2 V_{z0}^2}\right)\left(\frac{1}{\phi}\cos\phi - \frac{1}{\phi'}\cos\phi' - \frac{1}{\phi} + \frac{1}{\phi'}\right) - \frac{L D_3}{\phi V_{z0}}\left(\frac{L}{\phi'}\sin\phi' + \frac{L}{\phi'^2}\cos\phi' - \frac{L}{\phi'^2}\right)\right] \tag{11e}$$

$$\mathrm{Im}(P_{e3}) = \Gamma \nu k_c^2 \mu_0 \frac{\omega}{\gamma_0} \frac{L}{2-\beta_{\varphi 0}^2} \left[\left(\frac{A_3 L}{\phi V_{z0}} + \frac{B_3 L^2}{\phi^2 V_{z0}^2}\right)\left(\frac{1}{\phi}\sin\phi - \frac{1}{\phi'}\sin\phi'\right) - \frac{D_3}{\phi V_{z0}}\left(\frac{L^2}{\phi'^2}\sin\phi' - \frac{L^2}{\phi'}\cos\phi'\right)\right] \tag{11f}$$

$$\mathrm{Re}(P_{e4}) = \Gamma \nu \mu_0 k_c^2 \frac{\omega}{\gamma_0} \frac{L}{2-\beta_{\varphi 0}^2} \left[\left(\frac{A_4 L}{\phi' V_{z0}} + \frac{B_4 L^2}{\phi'^2 V_{z0}^2}\right)\left(\frac{1}{\phi}\cos\phi + \frac{1}{\phi}\cos\phi - \frac{1}{\phi} - \frac{1}{\phi}\right) + \frac{D_4}{\phi' V_{z0}}\left(\frac{L^2}{\phi}\sin\phi + \frac{L^2}{\phi^2}\cos\phi - \frac{L^2}{\phi^2}\right)\right] \tag{11g}$$

$$\mathrm{Im}(P_{e4}) = \Gamma \nu \mu_0 k_c^2 \frac{\omega}{\gamma_0} \frac{L}{2-\beta_{\varphi 0}^2} \left[\left(\frac{A_4 L}{\phi' V_{z0}} + \frac{B_4 L^2}{\phi'^2 V_{z0}}\right)\left(\frac{1}{\phi}\sin\phi - \frac{1}{\phi}\sin\phi\right) + \frac{D_4}{\phi' V_{z0}}\left(\frac{L^2}{\phi^2}\sin\phi - \frac{L^2}{\phi}\cos\phi\right)\right] \tag{11h}$$

式中，

$$\left.\begin{aligned} \phi &= \frac{\Omega L}{V_{z0}} \\ \phi' &= \frac{\Omega' L}{V_{z0}} \\ \phi &= 2k_{\parallel} L \end{aligned}\right\} \tag{12}$$

$$A_1 = -4(\omega - k_{/\!/}V_{z0})(Z_m'^2 + k_cR_0Z_m'Z_m'') + 2\beta_{\varphi0}^2[(\omega + k_{/\!/}V_{z0})Z_m'^2 + \omega k_cR_0Z_m'Z_m'') + \frac{\beta_{\varphi0}^4 k_{/\!/}V_{z0}Z_m'^2}{2-\beta_{\varphi0}^2} \tag{13a}$$

$$B_1 = [2\beta_{\varphi0}^2(\omega^2 - k_{/\!/}^2c^2) - 2\beta_{\varphi0}^2(\omega^2 - k_{/\!/}^2V_{z0}^2) - (\omega - k_{/\!/}V_{z0})(2-\beta_{\varphi0}^2)m\omega_c + \beta_{\varphi0}^4k_{/\!/}^2c^2]Z_m'^2 \tag{13b}$$

$$C_1 = \left\{\left[2\beta_{\varphi0}^2k_{/\!/}V_{z0}(\omega - m\omega_c) + (\omega - k_{/\!/}V_{z0})(2-\beta_{\varphi0}^2)m\omega_c\right]\frac{z}{V_{z0}} - (2-\beta_{\varphi0}^2)\beta_{\varphi0}^2c^2k_{/\!/}(\omega - m\omega_c)\frac{z}{V_{z0}^2}\right\}Z_m'^2 \tag{13c}$$

$$A_2 = -4(\omega + k_{/\!/}V_{z0})(Z_m'^2 + k_cR_0Z_m'Z_m'') + 2\beta_{\varphi0}^2[(\omega - k_{/\!/}V_{z0})Z_m'^2 + k_cR_0Z_m'Z_m''] - \frac{k_{/\!/}V_{z0}\beta_{\varphi0}^4Z_m'^2}{2-\beta_{\varphi0}^2} \tag{13d}$$

$$B_2 = [2\beta_{\varphi0}^2(\omega^2 + k_{/\!/}^2V_{z0}^2) + (\omega + k_{/\!/}V_{z0})m\omega_c(2-\beta_{\varphi0}^2) - 2\beta_{\varphi0}^2(\omega^2 - k_{/\!/}^2c^2) - \beta_{\varphi0}^4k_{/\!/}^2c^2]Z_m'^2 \tag{13e}$$

$$C_2 = \{-2\beta_{\varphi0}^2(\omega - k_{/\!/}V_{z0})(k_{/\!/}z - \Omega'z/V_{z0}) + [(2-\beta_{\varphi0}^2)k_{/\!/}^2V_{\varphi0}^2 + 2\beta_{\varphi0}^2\omega(k_{/\!/}V_{z0} - \Omega') - (2-\beta_{\varphi0}^2)m\omega_c\omega]z/V_{z0} + (2-\beta_{\varphi}^2)(m\omega_ck_{/\!/}z - k_{/\!/}\Omega'V_{\varphi0}^2z/V_{z0})\}Z_m'^2 \tag{13f}$$

$$A_3 = \omega\beta_{\varphi0}^2\left(2Z_m'^2 + 2k_cR_0Z_m'Z_m'' + Z_m'^2\frac{\beta_{\varphi0}^2}{2-\beta_{\varphi0}^2}\right) - 4(\omega - k_{/\!/}V_{z0}) \times\left[\left(1 - \frac{k_{/\!/}V_{z0}\beta_{\varphi0}^2}{2(\omega - k_{/\!/}V_{z0})}\right)Z_m'^2 + k_cR_0Z_m'Z_m'' + \frac{\beta_{\varphi0}^4Z_m'^2}{4(2-\beta_{\varphi0}^2)}\right] \tag{13g}$$

$$B_3 = \left[2\omega\beta_{\varphi0}^2k_{/\!/}V_{z0} - (2-\beta_{\varphi0}^2)V_{\varphi0}^2k_{/\!/}^2 - (2-\beta_{\varphi0}^2)(\omega - k_{/\!/}V_{z0})\left(k_{/\!/}V_{z0}\frac{2\beta_{\varphi0}^2}{2-\beta_{\varphi0}^2} + m\omega_c\right)\right]Z_m'^2 \tag{13h}$$

$$C_3 = \{2\omega\beta_{\varphi0}^2(k_{/\!/}z + \Omega z/V_{z0}) - (2-\beta_{\varphi0}^2)V_{\varphi0}^2k_{/\!/}(k_{/\!/}V_{z0} + \Omega)z/V_{z0} - (2-\beta_{\varphi0}^2) \times(\omega - k_{/\!/}V_{z0})[m\omega_c - (k_{/\!/}V_{z0} + \Omega)2\beta_{\varphi0}^2/(2-\beta_{\varphi0}^2)]z/V_{z0}^2\}Z_m'^2 \tag{13i}$$

$$A_4 = \omega\beta_{\varphi0}^2\left(2Z_m'^2 + 2k_cR_0Z_m'Z_m'' + Z_m'\frac{\beta_{\varphi0}^2}{2-\beta_{\varphi0}^2}\right) - 4(\omega + k_{/\!/}V_{z0}) \times\left[\left(1 + \frac{k_{/\!/}V_{z0}\beta_{\varphi0}^2}{2(\omega + k_{/\!/}V_{z0})}\right)Z_m' + k_cR_0Z_m'Z_m'' + \frac{\beta_{\varphi0}^4Z_m'^2}{4(2-\beta_{\varphi0}^2)}\right] \tag{13j}$$

$$B_4 = [2\beta_{\varphi0}^2\omega(\omega - k_{/\!/}V_{z0}) - 2\beta_{\varphi0}^2\omega^2 + 2V_{\varphi0}^2k_{/\!/}^2 - \beta_{\varphi0}^2V_{\varphi0}^2k_{/\!/} + (2-\beta_{\varphi0}^2) \times(\omega + k_{/\!/}V_{z0})(k_{/\!/}V_{z0}2\beta_{\varphi0}^2/(2-\beta_{\varphi0}^2) + m\omega_c)]Z_m'^2 \tag{13k}$$

$$C_4 = \left\{-2\omega\beta_{\varphi0}^2\left(k_{/\!/}z - \frac{\Omega'z}{V_{z0}}\right) + (2-\beta_{\varphi0}^2)(k_{/\!/}V_{z0} - \Omega')k_{/\!/}V_{\varphi0}^2\frac{z}{V_{z0}^2} + (2-\beta_{\varphi0}^2)(\omega + k_{/\!/}V_{z0})\left[m\omega_c + (k_{/\!/}V_{z0} - \Omega')\frac{2\beta_{\varphi0}^2}{2-\beta_{\varphi0}^2}\right]\frac{z}{V_{z0}}\right\}Z_m'^2 \tag{13l}$$

$$D_1 = \left\{[2\beta_{\varphi0}^2k_{/\!/}V_{z0}(\omega - m\omega_c) + (\omega - k_{/\!/}V_{z0})(2-\beta_{\varphi0}^2)m\omega_c]\frac{1}{V_{z0}}\right.$$

$$-\left[(2-\beta_{\varphi 0}^{2})\beta_{\varphi 0}^{2}c^{2}k_{\parallel}(\omega-m\omega_{c})\right]\frac{1}{V_{z0}}\Big\}Z_{m}^{\prime 2} \tag{13m}$$

$$D_2=\Big\{(\omega-k_{\parallel}V_{z0})2\beta_{\varphi 0}^{2}\left(k_{\parallel}-\frac{\Omega'}{V_{z0}}\right)+\beta_{\varphi 0}^{2}[k_{\parallel}^{2}c^{2}(2-\beta_{\varphi 0}^{2})+2(k_{\parallel}V_{z0}-\Omega')\omega-m\omega_{c}\omega]$$

$$\times\frac{1}{V_{z0}}+(2-\beta_{\varphi 0}^{2})k_{\parallel}\Omega'\beta_{\varphi 0}^{2}c^{2}\frac{1}{V_{z0}^{2}}\Big\}Z_{m}^{\prime 2} \tag{13n}$$

$$\left.\begin{aligned}&\nu=\frac{e^{2}n_{l}}{c^{2}\varepsilon_{0}m_{0}},\ \text{一无量纲常数}\\&D_3=D_1,\ D_4=D_2\end{aligned}\right\} \tag{13o}$$

3. 数值计算

考虑一个实际同轴腔(实验工作中所采用的腔体)，参数和结果如图 1 和图 2 所示。

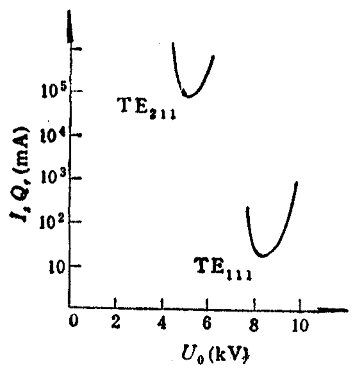

图 1 不同模式的起振电流

$R_0=1.2\text{mm}, R_a=0.1\text{mm}$

$R_b=40\text{mm}, L=200\text{mm}$

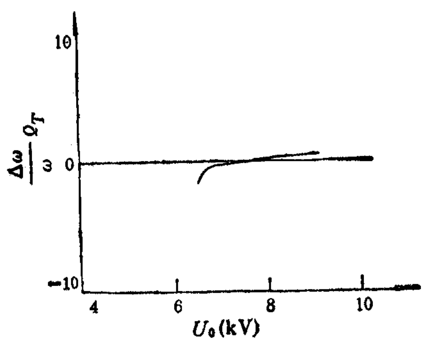

图 2 静电聚束场与频偏的关系

TE_{111}, $R_0=1.2\text{mm}, R_a=0.1\text{mm}$

$R_b=40\text{mm}, L=200\text{mm}$

从图可以看出，对应于较低次模式（TE_{111} 模)的起振电流（$I_{st}Q_T=22.8\text{mA}$）比高次模式的（TE_{211}，$I_{st}Q_T=6.44\times10^4\text{mA}$）要低得多。这一结果同静磁单腔管是类似的[3]。

本文中求得的不同模式的起振电流和频率偏移曲线，对静电电子迴旋脉塞机理验证实验提供了理论数据。同时，对实验测试中工作模式的建立和判别，也有指导作用。

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STUDY OF THE STARTING CURRENT AND FREQUENCY DEVIATION OF ELECTROSTATIC GYROMONOTRON

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Abstract From relativistic linear Vlasov-Maxwell equations, the representations of the starting current and frequency deviation of a electrostatic gyromonotron with a field distribution $\sin k_{\parallel} z$ are derived and the numerical calculation are carried out.

Key words Electrostatic electron cyclotron resonance maser; Electrostatic gyromonotron; Starting current; Frequency deviation