第12卷 第6期
1990年11月

电子科学学刊
JOURNAL OF ELECTRONICS

Vol. 12, No. 6
Nov., 1990

波导混合模问题的修正边界元法分析*

洪 伟
(东南大学，南京)

摘要 本文建立了分析波导混合模问题的修正边界元模型；导出了关于混合模的耦合边界积分方程组及其退化形式，并用矩量法将之离散为齐次代数方程组，从而得波导的混合模传输常数；最后以部分介质填充波导为例进行了计算，所得结果与横向谐振法结果吻合得很好.

关键词 波导;混合模;传输线;边界元法;色散特性

一、引 言

边界元法（BEM）源于力学领域，具有降低所研究边值问题维数的特性，近几年来在电磁领域中也越来越受到人们的重视. 虽然边界元法已将三维边值问题化成了二维问题，二维化成了一维，但若边界尺度较大时，离散化后的代数方程组阶数仍然很高，从而计算量很大. 为此，作者在文献[1]中提出了修正边界元法，其基本思想是把边界分成规则边界和不规则边界两部分，采用满足规则边界条件的修正 Green 函数作为基本解，将边界积分方程压缩到剩余的不规则边界上，这样分段和计算仅需在不规则边界上进行，从而大大地减小了计算量. 对若干 TEM 模及单模传输线的分析结果表明，该方法准确有效[2]. 在文献[3]中我们将这种方法用于分析波导不连续性问题. 同样获得了满意的效果.

传输线的全波分析（或混合模分析）是电磁场边值问题中难度较大而又很有实用价值的问题. 本文建立的混合模的修正边界元模型为混合模传输线的分析提供了一有效途径.

二、混合模场和边界条件

在填充各向同性、均匀和无耗媒质区域 Ω 中电场 $\boldsymbol{E}$ 和磁场 $\boldsymbol{H}$ 可用赫兹位 $\boldsymbol{\Pi}^e$ 和 $\boldsymbol{\Pi}^h$ 表示如下：

$$\boldsymbol{E}=k^2\boldsymbol{\Pi}^e+\nabla\nabla\cdot\boldsymbol{\Pi}^e-j\omega\mu\nabla\times\boldsymbol{\Pi}^h \tag{1}$$

$$\boldsymbol{H}=k^2\boldsymbol{\Pi}^h+\nabla\nabla\cdot\boldsymbol{\Pi}^h+j\omega\varepsilon\nabla\times\boldsymbol{\Pi}^e \tag{2}$$

式中 $\boldsymbol{\Pi}^e$ 和 $\boldsymbol{\Pi}^h$ 满足 Hemholtz 方程

* 1989年6月23日收到，1990年3月5日修改定稿.

$$(\nabla^2 + k^2)\boldsymbol{\Pi}^{e,h} = 0 \tag{3}$$

对于导波问题,令 $\boldsymbol{\Pi}^{e,h} = \hat{z}\Pi^{e,h}(x, y)e^{-j\beta z}$, 即得关于 $\hat{z}$ 的 TE 模和 TM 模, 这里 β 为传播常数. (1) 和 (2) 式中的 $\boldsymbol{E}$ 和 $\boldsymbol{H}$ 为 TE 和 TM 模的迭加,即混合模场, 写成分量形式如下(参见图 1 坐标系):

$$E_n = -j\beta\frac{\partial\Pi^e}{\partial n} - j\omega\mu\frac{\partial\Pi^h}{\partial\tau} \tag{4a}$$

$$E_\tau = -j\beta\frac{\partial\Pi^e}{\partial\tau} + j\omega\mu\frac{\partial\Pi^h}{\partial n} \tag{4b}$$

$$E_z = k_c^2\Pi^e \tag{4c}$$

$$H_n = -j\beta\frac{\partial\Pi^h}{\partial n} + j\omega\varepsilon\frac{\partial\Pi^e}{\partial\tau} \tag{4d}$$

$$H_\tau = -j\beta\frac{\partial\Pi^h}{\partial\tau} - j\omega\varepsilon\frac{\partial\Pi^e}{\partial n} \tag{4e}$$

$$H_z = k_c^2\Pi^h \tag{4f}$$

式中 $k_c^2 = k^2 - \beta^2 = \varepsilon_r\mu_r k_0^2 - \beta^2$ 为截止波数.

由上面场分量可导出位 $\Pi^{e,h}$ 的边界条件. 在电壁上:

$$\Pi^e = 0,\ \frac{\partial\Pi^e}{\partial\tau} = 0, \frac{\partial\Pi^h}{\partial n} = 0 \tag{5}$$

在磁壁上:

$$\Pi^h = 0,\ \frac{\partial\Pi^h}{\partial\tau} = 0,\ \frac{\partial\Pi^e}{\partial n} = 0 \tag{6}$$

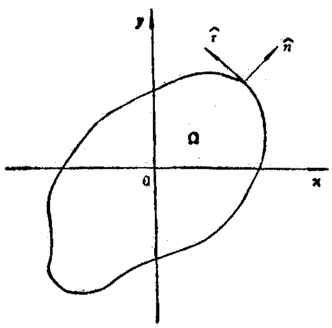

图 1 任意截面波导

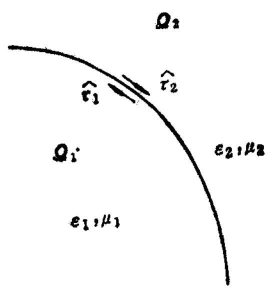

图 2 两种媒质交界面

在两种媒质分界面上(图 2), 有连续性条件

$$k_{c1}^c\Pi_1^e = k_{c2}^2\Pi_2^e \tag{7}$$

$$k_{c1}^2\Pi_1^h = k_{c2}^2\Pi_2^h \tag{8}$$

和

$$-\beta\frac{\partial\Pi_1^e}{\partial\tau_1} + \omega\mu_1\frac{\partial\Pi_1^h}{\partial n_1} = \beta\frac{\partial\Pi_2^e}{\partial\tau_2} - \omega\mu_2\frac{\partial\Pi_2^h}{\partial n_2}$$

$$-\beta\frac{\partial\Pi_1^h}{\partial\tau_1} - \omega\varepsilon_1\frac{\partial\Pi_1^e}{\partial n_1} = \beta\frac{\partial\Pi_2^h}{\partial\tau_2} + \omega\varepsilon_2\frac{\partial\Pi_2^e}{\partial n_2}$$

将 (7)、(8) 式代入上面两式得

$$\frac{\partial \Pi_2^h}{\partial n_2} = -\frac{\mu_1}{\mu_2}\frac{\partial \Pi_1^h}{\partial n_1} + \frac{\beta}{\omega\mu_2}\frac{k_{c2}^2 - k_{c1}^2}{k_{c2}^2}\frac{\partial \Pi_1^e}{\partial \tau_1} \tag{9}$$

$$\frac{\partial \Pi_2^e}{\partial n_2} = -\frac{\varepsilon_1}{\varepsilon_2}\frac{\partial \Pi_1^e}{\partial n_1} + \frac{\beta}{\omega\varepsilon_2}\frac{k_{c1}^2 - k_{c2}^2}{k_{c2}^2}\frac{\partial \Pi_1^h}{\partial \tau_1} \tag{10}$$

三、耦合边界积分方程

由(3)式知 $\Pi^{e,h}$ 满足二维标量 Hemholtz 方程

$$(\nabla_\perp^2 + k_{c2}^2)\Pi^{e,h} = 0 \tag{11}$$

对应的 Green 函数 $G^{e,h}$ 满足

$$(\nabla_\perp^2 + k_c^2)G^{e,h} = -\delta(\boldsymbol{R} - \boldsymbol{R}_s) \tag{12}$$

式中 $\boldsymbol{R}$ 和 $\boldsymbol{R}_s$ 分别表示场点和源点的位置矢量. 对于边界法场点和源点都位于边界上,如图 3 所示.用一半径为 σ 的小圆弧绕过场点 $\boldsymbol{R}$,然后令 σ 趋于零,则在 Ω 内 $G^{e,h}$ 满足齐次 Hemholtz 方程

$$(\nabla_\perp^2 + k_c^2)G^{e,h} = 0 \tag{13}$$

用 $G^{e,h}$ 和 $\Pi^{e,h}$ 分别对(11)和(13)式两边作内积,然后相减,并利用 Green 第二恒等式

$$\int_\Omega (G^{e,h}\nabla_\perp^2\Pi^{e,h} - \Pi^{e,h}\nabla_\perp^2 G^{e,h})d\Omega = \oint_{\partial\Omega}\left(G^{e,h}\frac{\partial \Pi^{e,h}}{\partial n_s} - \Pi^{e,h}\frac{\partial G^{e,h}}{\partial n_s}\right)d\Gamma_s \tag{14}$$

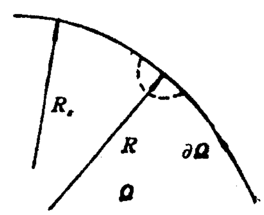

图 3　奇异点的处理

得耦合边界积分方程

$$\oint_{\partial\Omega}\left(G^{e,h}\frac{\partial \Pi^{e,h}}{\partial n_s} - \Pi^{e,h}\frac{\partial G^{e,h}}{\partial n_s}\right)d\Gamma_s = 0 \tag{15}$$

式中下标"s"表示源点.

四、两种介质填充波导

因为多种介质填充波导仅是两种介质填充波导的简单推广,因此下面仅就如图 4 所示两种介质填充波导建立其修正边界元模型.

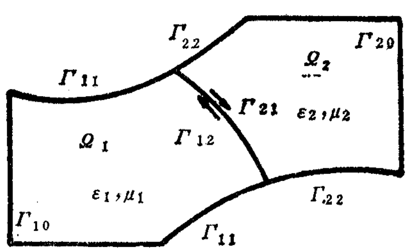

图 4　两种介质填充波导

在图 4 中, $\partial\Omega_1 = \Gamma_{10} + \Gamma_{11} + \Gamma_{12}$, $\partial\Omega_2 = \Gamma_{20} + \Gamma_{21} + \Gamma_{22}$; Γ_{10}, Γ_{11}, Γ_{20}, Γ_{22} 为电壁或磁壁, Γ_{10}, Γ_{20} 为规则边界部分. 将 $\partial\Omega_1$ 和 $\partial\Omega_2$ 上的耦合边界积分方程联立可得如下边界积分方程组:

$$\sum_{i=0}^{2}\int_{\Gamma_{1i}}\left(G_1^e\frac{\partial \Pi_1^e}{\partial n_1} - \Pi_1^e\frac{\partial G_1^e}{\partial n_1}\right)d\Gamma_s = 0 \tag{16a}$$

$$\sum_{i=0}^{2}\int_{\Gamma_{2i}}\left(G_2^e\frac{\partial \Pi_2^e}{\partial n_2} - \Pi_2^e\frac{\partial G_2^e}{\partial n_2}\right)d\Gamma_s = 0 \tag{16b}$$

$$\sum_{i=0}^{2}\int_{\Gamma_{1i}}\left(G_1^h\frac{\partial\Pi_1^h}{\partial n_1}-\Pi_1^h\frac{\partial G_1^h}{\partial n_1}\right)d\Gamma_s=0 \tag{16c}$$

$$\sum_{i=0}^{2}\int_{\Gamma_{2i}}\left(G_2^h\frac{\partial\Pi_2^h}{\partial n_2}-\Pi_2^h\frac{\partial G_2^h}{\partial n_2}\right)d\Gamma_s=0 \tag{16d}$$

令 $G_i^{e,h}$ 在 $\Gamma_{i0}(i=1,2)$ 上满足与 $\Pi_i^{e,h}$ 相同的边界条件(见(5),(6)式),则上面方程组在 Γ_{i0} 上的积分为零,从而退化为

$$\int_{\Gamma_{ii}}\left(G_i^{e,h}\frac{\partial\Pi_i^{e,h}}{\partial n_i}-\Pi_i^{e,h}\frac{\partial G_i^{e,h}}{\partial n_i}\right)d\Gamma_s+\int_{\Gamma_{12}}\left(G_i^{e,h}\frac{\partial\Pi_i^{e,h}}{\partial n_i}-\Pi_i^{e,h}\frac{\partial G_i^{e,h}}{\partial n_i}\right)d\Gamma_s=0,\quad(i=1,2) \tag{17}$$

不失一般性,不妨设 Γ_{11} 为电壁,Γ_{22} 为磁壁,利用边界条件(5)、(6)式和两种介质交接面上的连续性条件(7)—(10)式,可将(17)式化为

$$\begin{bmatrix}\int_{\Gamma_{11}}G_1^e q_1^e d\Gamma_s\\ -\int_{\Gamma_{22}}\frac{\partial G_2^e}{\partial n_2}\Pi_2^e d\Gamma_s\\ -\int_{\Gamma_{11}}\frac{\partial G_1^h}{\partial n_1}\Pi_1^h d\Gamma_s\\ \int_{\Gamma_{22}}G_2^h q_2^h d\Gamma_s\end{bmatrix}+\int_{\Gamma_{12}}[T]\begin{bmatrix}q_1^e\\ \Pi_1^e\\ q_1^h\\ \Pi_1^h\end{bmatrix}d\Gamma_s=0 \tag{18}$$

其中 $q_i^{e,h}=\dfrac{\partial\Pi_i^{e,h}}{\partial n_i}$,$(i=1,2)$,算子矩阵 $[T]$ 为

$$[T]=\begin{bmatrix}G_1^e & -\dfrac{\partial G_1^e}{\partial n_1} & 0 & 0\\ -\dfrac{\varepsilon_1}{\varepsilon_2}G_2^e & -\dfrac{k_{c1}^2}{k_{c2}^2}\dfrac{\partial G_2^e}{\partial n_2} & 0 & \dfrac{\beta(k_{c1}^2-k_{c2}^2)}{\omega\varepsilon_2k_{c2}^2}G_2^e\dfrac{\partial}{\partial\tau_1}\\ 0 & 0 & G_1^h & -\dfrac{\partial G_1^h}{\partial n_1}\\ 0 & \dfrac{\beta(k_{c2}^2-k_{c1}^2)}{\omega\mu_2k_{c2}^2}G_2^h\dfrac{\partial}{\partial\tau_1} & -\dfrac{\mu_1}{\mu_2}G_2^h & -\dfrac{k_{c1}^2}{k_{c2}^2}\dfrac{\partial G_2^h}{\partial n_2}\end{bmatrix} \tag{19}$$

五、离 散 化

用基函数 $\phi_n^{e,h}$、$\phi_n^{e,h}(n=1,2,\cdots,N)$ 的线性组合近似 $q_1^{e,h}$ 和 $\Pi_1^{e,h}$,即

$$q_1^{e,h}=\sum_{n=1}^{N}a_n^{e,h}\phi_n^{e,h}(\Gamma_s) \tag{20}$$

$$\Gamma_s\in\Gamma_{12}$$

$$\Pi_1^{e,h}=\sum_{n=1}^{N}b_n^{e,h}\phi_n^{e,h}(\Gamma_s) \tag{21}$$

在 Γ_{11} 和 Γ_{22} 上,用 $\xi_n^{e,h}$ 和 $\eta_n^{e,h}$ 的线性组合近似 q_1^e、q_2^h 和 Π_2^e、Π_1^h 如下:

$$q_1^e(\Gamma_s) = \sum_{n=1}^{N_1} c_n^e \xi_n^e(\Gamma_s) \tag{22}$$

$$\Gamma_s \in \Gamma_{11}$$

$$\Pi_1^h(\Gamma_s) = \sum_{n=1}^{N_1} d_n^h \eta_n^h(\Gamma_s) \tag{23}$$

$$q_2^h(\Gamma_s) = \sum_{n=1}^{N_1} c_n^h \xi_n^h(\Gamma_s) \tag{24}$$

$$\Gamma_s \in \Gamma_{22}$$

$$\Pi_2^e(\Gamma_s) = \sum_{n=1}^{N_1} d_n^e \eta_n^e(\Gamma_s) \tag{25}$$

将(20)—(25)式代入边界积分方程组(18)，然后用权函数

$$u_m^{e,h}(\Gamma_s), \quad \Gamma_s \in \Gamma_{11} + \Gamma_{12}, \quad m = 1, 2, \cdots, N + N_1 \tag{26}$$

$$v_m^{e,h}(\Gamma_s), \quad \Gamma_s \in \Gamma_{22} + \Gamma_{12}, \quad m = 1, 2, \cdots, N + N_2 \tag{27}$$

对两边作内积得线性代数方程组

$$\begin{bmatrix} [A]_{11} & & & & [B]_{11} & [B]_{12} & & \\ & [A]_{22} & & & [B]_{21} & [B]_{22} & & [B]_{24} \\ & & [A]_{33} & & & & [B]_{33} & [B]_{34} \\ & & & [A]_{44} & & [B]_{42} & [B]_{43} & [B]_{44} \end{bmatrix} \begin{bmatrix} \boldsymbol{c}_e \\ \boldsymbol{d}_e \\ \boldsymbol{d}_h \\ \boldsymbol{c}_h \\ \boldsymbol{a}_e \\ \boldsymbol{b}_e \\ \boldsymbol{a}_h \\ \boldsymbol{b}_h \end{bmatrix} = [\mathbf{0}] \tag{28}$$

若以上角标"t"表示转置，则

$$\boldsymbol{C}_e^t = [c_1^e, c_2^e, \cdots, c_{N_1}^e] \tag{29}$$

其它列矢量定义类似．$[\boldsymbol{A}_{ij}]$ 和 $[\boldsymbol{B}_{ij}]$ 的元素见附录．将(28)简记为

$$[\boldsymbol{D}]\ [\mathbf{c}] = [\mathbf{0}] \tag{30}$$

则该方程有非零解的条件为

$$\det[D] = 0 \tag{31}$$

由此即可求得传播常数 β.

六、数 值 例

下面以图5所示部分介质填充波导为例说明上面所建模型的正确性．图中 $x = 0$, a 及 $y = 0, b$ 处为电壁．令 Green 函数 $G_i^{e,h}(i = 1, 2)$ 满足如下定解问题：

$$\nabla_\perp^2 G_i^e + k_{ci}^2 G_i^e = -\delta(x - x_s)\delta(y - y_s) \tag{32}$$

$$G_i^e|_{x=0,a} = 0, \quad G_i^e|_{y=0,b} = 0, \quad (i = 1, 2)$$

$$\nabla_\perp^2 G_i^h + k_{ci}^2 G_i^h = -\delta(x - x_s)\delta(y - y_s) \tag{33}$$

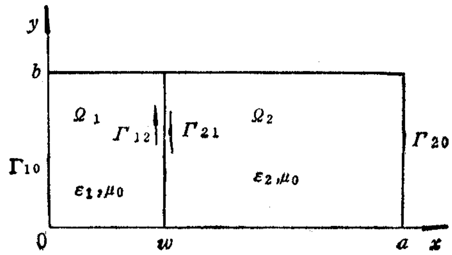

图5 部分介质填充波导

$$\left.\frac{\partial G_i^h}{\partial y}\right|_{y=0,b}=0,\qquad \left.\frac{\partial G_i^h}{\partial x}\right|_{x=0,a}=0,\qquad (i=1,2)$$

则边界积分方程组(18)式进一步简化为

$$\int_{\Gamma_{12}}[T]\begin{bmatrix}q_1^e\\ \Pi_1^e\\ q_1^h\\ \Pi_1^h\end{bmatrix}d\Gamma_s=0 \tag{34}$$

对应的代数方程组(28)式变为

$$\begin{bmatrix}[B]_{11} & [B]_{12} & & \\ [B]_{21} & [B]_{22} & & [B]_{24}\\ & & [B]_{33} & [B]_{34}\\ & [B]_{42} & [B]_{43} & [B]_{44}\end{bmatrix}\begin{bmatrix}\boldsymbol{a}_e\\ \boldsymbol{b}_e\\ \boldsymbol{a}_h\\ \boldsymbol{b}_h\end{bmatrix}=[\boldsymbol{0}] \tag{35}$$

解定解问题(32)、(33)式得

$$G_i^e=\sum_{n=1}^{\infty}\frac{2}{b\gamma_{ni}\sin\gamma_{ni}a}\sin\frac{n\pi y}{b}\sin\frac{n\pi y_s}{b}\begin{cases}\sin\gamma_{ni}x\sin\gamma_{ni}(a-x_s)\\ \qquad 0\leqslant x\leqslant x_s\\ \sin\gamma_{ni}x_s\sin\gamma_{ni}(a-x)\\ \qquad x_s\leqslant x\leqslant a\end{cases} \tag{36}$$

$$G_i^h=\sum_{n=0}^{\infty}\frac{-2}{\alpha_n b\gamma_{ni}\sin\gamma_{ni}a}\cos\frac{n\pi y}{b}\cos\frac{n\pi y_s}{b}\begin{cases}\cos\gamma_{ni}x\cos\gamma_{ni}(a-x_s)\\ \qquad 0\leqslant x\leqslant x_s\\ \cos\gamma_{ni}x_s\cos\gamma_{ni}(a-x)\\ \qquad x_s\leqslant x\leqslant a\end{cases} \tag{37}$$

$$\gamma_{ni}^2=k_{ci}^2-\left(\frac{n\pi}{b}\right)^2=\varepsilon_i k_0^2-\beta^2-\left(\frac{n\pi}{b}\right)^2$$

$$i=1,2;\qquad \alpha_n=\begin{cases}2, & n=0\\ 1, & n>0\end{cases}$$

式中 (x_s, y_s) 为源点坐标,(x, y) 为场点坐标。

因为在积分路径 Γ_{12} 上 $x_s=w$ 为常数,故 $\phi_n^{e,h}$ 和 $\phi_n^{e,h}$ 仅为 y_s 的函数。取基函数

等于权函数(即 Galerkin 法)如下：

$$\left.\begin{aligned} u_n^e &= \phi_n^e = \sin\frac{n\pi y_s}{b} \\ v_n^e &= \phi_n^e = \sin\frac{n\pi y_s}{b} \\ u_n^h &= \phi_n^h = \cos\frac{n\pi y_s}{b} \\ v_n^h &= \phi_n^h = \cos\frac{n\pi y_s}{b} \end{aligned}\right\} \tag{38}$$

这些函数都满足 $y_s = 0, b$ 上的边界条件。

将 (36)—(38) 式代入附录中 $[B]_{ij}$ 各元素的表达式即可证明 $[B]_{ij}$ 为对角矩阵，且对角线元素为不含奇点的解析表达式。

在矩阵方程 (35) 式中消去 $\boldsymbol{a}_e$ 和 $\boldsymbol{a}_h$ 得

$$([B]_{22} - [B]_{21}[B]_{11}^{-1}[B]_{12})\boldsymbol{b}_e + [B]_{24}\boldsymbol{b}_h = \boldsymbol{0}$$
$$[B]_{42}\boldsymbol{b}_e + ([B]_{44} - [B]_{43}[B]_{33}^{-1}[B]_{34})\boldsymbol{b}_h = \boldsymbol{0}$$

再消去 $\boldsymbol{b}_h$ 得

$$\{[B]_{24}^{-1}([B]_{22} - [B]_{21}[B]_{11}^{-1}[B]_{12}) - ([B]_{44} - [B]_{43}[B]_{33}^{-1}[B]_{34})^{-1}[B]_{42}\}\boldsymbol{b}_e = \boldsymbol{0} \tag{39}$$

简记

$$[D]\boldsymbol{b}_e = \boldsymbol{0} \tag{40}$$

由于 $[B]_{ij}$ 皆为对角阵，因此 $[D]$ 也是对角阵。(40) 式有非零解的条件为

$$\det[D] = 0 \tag{41}$$

由此就可获得部分介质填充波导的色散特性。

图 6 和图 7 分别给出了 E 面和 H 面部分介质填充矩形波导色散特性的数值计算结果，与横向谐振法结果[4]吻合得很好。

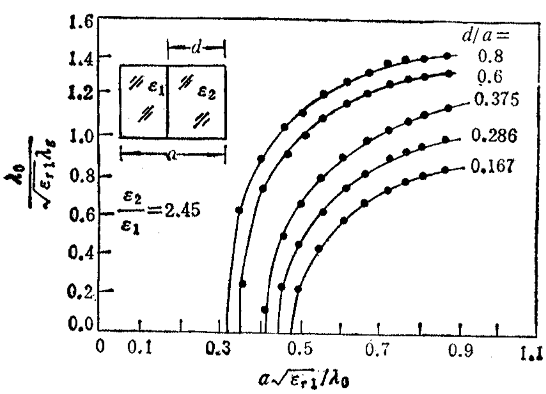

图 6　E 面两种介质填充波导的色散特性曲线
—— 文献[4]　●●● 本文方法

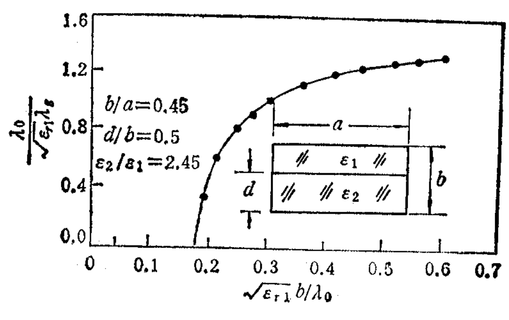

图 7　H 面两种介质填充波导的色散特性曲线
—— 文献[4]　●●● 本文方法

对于上面的例子，若用 BEM，则需在整个边界 $\Gamma_{11} + \Gamma_{22} + \Gamma_{12}$ 上分段；而本文方法只需在 Γ_{12} 上进行，计算量约为 BIM 的十分之一左右。

由上面分析可知，本文模型很容易用于分析微带类、鳍线（Finline）类和介质波导等传输线的色散特性。

感谢李嗣范教授的指导。

附 录

矩阵元素(见表 1)

表 1

$$A_{11}^{mn}=\int_{\Gamma_{11}+\Gamma_{12}}\int_{\Gamma_{11}} G_1^e(\Gamma,\Gamma_s)u_m^e(\Gamma)\xi_n^e(\Gamma_s)d\Gamma_s d\Gamma$$

$$A_{22}^{mn}=-\int_{\Gamma_{22}+\Gamma_{21}}\int_{\Gamma_{22}} \frac{\partial}{\partial n_2} G_2^e(\Gamma,\Gamma_s)\nu_m^e(\Gamma)\eta_n^e(\Gamma_s)d\Gamma_s d\Gamma$$

$$A_{33}^{mn}=-\int_{\Gamma_{11}+\Gamma_{12}}\int_{\Gamma_{11}} \frac{\partial}{\partial n_1} G_1^h(\Gamma,\Gamma_s)u_m^h(\Gamma)\eta_n^h(\Gamma_s)d\Gamma_s d\Gamma$$

$$A_{44}^{mn}=\int_{\Gamma_{22}+\Gamma_{21}}\int_{\Gamma_{22}} G_2^h(\Gamma,\Gamma_s)\nu_m^h(\Gamma)\xi_n^h(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{11}^{mn}=\int_{\Gamma_{11}+\Gamma_{12}}\int_{\Gamma_{12}} G_1^e(\Gamma,\Gamma_s)u_m^e(\Gamma)\psi_n^e(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{12}^{mn}=-\int_{\Gamma_{11}+\Gamma_{12}}\int_{\Gamma_{12}} \frac{\partial}{\partial n_1} G_1^e(\Gamma,\Gamma_s)u_m^e(\Gamma)\phi_n^e(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{21}^{mn}=-\frac{\varepsilon_1}{\varepsilon_2}\int_{\Gamma_{22}+\Gamma_{21}}\int_{\Gamma_{21}} G_2^e(\Gamma,\Gamma_s)\nu_m^e(\Gamma)\psi_n^e(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{22}^{mn}=-\frac{k_{c1}^2}{k_{c2}^2}\int_{\Gamma_{22}+\Gamma_{21}}\int_{\Gamma_{12}} \frac{\partial}{\partial n} G_2^e(\Gamma,\Gamma_s)\nu_m^e(\Gamma)\phi_n^e(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{24}^{mn}=\frac{\beta(k_{c1}^2-k_{c2}^2)}{\omega\varepsilon_2 k_{c2}^2}\int_{\Gamma_{22}+\Gamma_{21}}\int_{\Gamma_{21}} G_2^e(\Gamma,\Gamma_s)\nu_m^e(\Gamma)\frac{d}{d\Gamma}\phi_n^h(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{33}^{mn}=\int_{\Gamma_{11}+\Gamma_{12}}\int_{\Gamma_{12}} G_1^h(\Gamma,\Gamma_s)u_m^h(\Gamma)\psi_n^h(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{34}^{mn}=-\int_{\Gamma_{11}+\Gamma_{12}}\int_{\Gamma_{12}} \frac{\partial}{\partial n_1} G_1^h(\Gamma,\Gamma_s)u_m^h(\Gamma)\phi_n^h(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{42}^{mn}=\frac{\beta(k_{c2}^2-k_{c1}^2)}{\omega\mu_2 k_{c2}^2}\int_{\Gamma_{22}+\Gamma_{21}}\int_{\Gamma_{12}} G_2^h(\Gamma,\Gamma_s)\nu_m^h(\Gamma)\frac{d}{d\Gamma_s}\phi_n^e(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{43}^{mn}=-\frac{\mu_1}{\mu_2}\int_{\Gamma_{22}+\Gamma_{21}}\int_{\Gamma_{12}} G_2^h(\Gamma,\Gamma_s)\nu_m^h(\Gamma)\psi_n^h(\Gamma_s)d\Gamma_s d\Gamma$$

$$B_{44}^{mn}=-\frac{k_{c1}^2}{k_{c2}^2}\int_{\Gamma_{22}+\Gamma_{21}}\int_{\Gamma_{12}} \frac{\partial}{\partial n_2} G_2^h(\Gamma,\Gamma_s)\nu_m^h(\Gamma)\phi_n^h(\Gamma_s)d\Gamma_s d\Gamma$$

参 考 文 献

[1] 洪伟，李嗣范，修正边界元法及其在电磁场边值问题中的应用，电子学报，1987年，第6期，第49—54页.

[2] Hong Wei, Analysis of Waveguide Discontinuities by A Modified Boundary Element Method, Proc. of ISAP, August, 1989, Japan, pp. 1037—1040.

[3] Zhang Wenxun, Hong Wei, Yuan Jinyun, Computation of the Characteristic Impedances for the Irregular Transmission Lines by Using a Modified BEM, Proc. of OFSET, May 1987, Nanjing, China, pp. 415—420.

[4] N. Marcuvitz, Waveguide Handbook, McGraw-Hill, N. Y., 1951.

ANALYSIS OF HYBRID MODES OF WAVEGUIDE BY A MODIFIED BOUNDARY ELEMENT METHOD

Hong Wei
(Southeast University, Nanjing)

Abstract A modified boundary element method for analysing hybrid modes of waveguide is presented. The coupled boundary integral equations and their reduced forms are deduced for hybrid mode problems, and these equations are discreted into a system of linear algebraic equations by moment method. Finally the dispersion characteristics of rectangular waveguide filled with two kinds of dielectrics are calculated and the numerical results are in good agreement with that obtained by transverse resonating method.

Key words Waveguide; Hybrid mode; Transmission line; Boundary element method; Dispersion characteristics