一种空间相关高斯噪声背景下的时变时延估计算法 1

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摘 要 在空间相关高斯噪声的背景下,基于二阶统计量的时延估计方法会失效,该文提出了一种基于三阶统计量的自适应时变时延估计算法,并分析了算法的收敛性,最后的仿真结果表明该算法可以有效地抑制相关高斯噪声.
 关键词 时延估计,三阶统计量
 中图号 TN911.23

1引言

实际情况下,由于有限长度的数据记录以及噪声并非完全独立,常常导致基于二阶统计量的时延估计方法的失效。当声源为非高斯信号,加性噪声为空间相关的零均值高斯噪声时,可以在高阶累量域获得良好的参数估计效果,这是因为高斯过程高于二阶的累积量恒等于零。当使用高阶累积量作为分析工具时,理论上可以完全抑制高斯噪声的影响。在这一领域,提出了若干处理方法,这些方法包括频域及时域的方法^[1,2]。C.L.Nikias 和 Renlong Pan 提出了一类基于双谱的时延估计算法.频域方法要求计算接收信号的双谱或互双谱.时域的方法是一个参数化的估计方法,计算量小于频域法^[3]。J. Hinich 和 G. R. Wilson 分析了影响基于双谱相位的时延估计算法性能的若干因素,指出性能是样本分析长度、信号双相干系数、信噪比及噪声相关性的函数^[4]。由于 C.L.Nikias 和 Renlong Pan 提出的基于三阶累积量的时域估计算法,涉及到矩阵求逆,在一些条件下,由于矩阵是病态的,从而导致时延的不可辨识。J.K. Tugnait提出了四阶累量域的一系列时域估计算法^[5-7],这些方法均不要求矩阵求逆。H.H. Chiang 和 C. L. Nikias 又提出了基于三阶累积量的自适应时延估计算法^[8],不涉及矩阵求逆,且具有跟踪时变时延的能力。同时,基于奇数阶的累量方法,还具有抑制对称概率密度函数噪声的能力。然而,高阶累量域的算法的一个缺点是,要求比二阶累量域更高的数据长度,且阶数越高,要求的数据长度越长。综合以上因素,本文提出了一种三阶累量域的时延估计算法。

2 三阶累量域的自适应时延直接估计算法

考虑两路接收到的离散时间信号:

$$\left. \begin{array}{l} x_1(k) = s(k) + n_1(k) \\ x_2(k) = \alpha \cdot s(k - D_{12}(k)) + n_2(k) \end{array} \right\}$$
(1)

式中 *s* 是信号, α 是两路信号传播起伏之比, n_1, n_2 是观测噪声,包括测量噪声和环境噪声, D_{12} 是待估计的时延。对于平稳情形有 $D_{12}(k) = \hat{D}$,且有如下关系式成立:

$$x_2(k) = \sum_{i=-\infty}^{\infty} a_i x_1(k-i) - n_1(k-\hat{D}) + n_2(k)$$
(2)

1 2001-10-26 收到, 2002-04-29 改回

理论上, $a_i = 0$, 对于所有的 *i*, 除了 $i = \hat{D}$, $a_{\hat{D}} = 1$. 实际过程中, \hat{D} 总是有限的, 因而有

$$x_2(k) = \sum_{i=-P}^{P} a_i x_1(k-i) - n_1(k-\hat{D}) + n_2(k)$$
(3)

P 的取值大于可能的最大时延值。两边同乘以 $x_1(k + \tau)x_2(k + \rho)$, 且取期望:

$$R_{x_2x_1x_1}(\tau,\rho) = \sum_{i=-P}^{P} a_i R_{x_1x_1x_1}(\tau+i,\rho+i) - R_{n_1x_1x_1}(\tau+\hat{D},\rho+\hat{D}) + R_{n_2x_1x_1}(\tau,\rho)$$
(4)

为了实现自适应的时延估计方案,需要首先定义一个最优准则. 假定 $R_{x_1x_1x_1}(\tau, \rho)$ 是 $\{x_1(k)\}$ 的三阶累积量. 那么,如图 1 所示,系数为 a_i 的 FIR 滤波器的输出为

$$R_{zx_1x_1}(\tau,\rho) = \sum_{i=-P}^{P} a_i R_{x_1x_1x_1}(\tau+i,\rho+i)$$
(5)



图 1 三阶累量域的自适应时延估计算法原理框图

最佳准则定义为期望输出 R_{x2x1x1} 与实际输出 R_{zx1x1} 的平方误差之和:

$$\xi = \sum_{\tau} \sum_{\rho} [R_{x_2 x_1 x_1}(\tau, \rho) - R_{z x_1 x_1}(\tau, \rho)]^2$$
(6)

选择不同的 τ , ρ 可得到不同的准则,本文令 $\tau = \rho = -P, -P + 1, \dots, P$,最优准则成为

$$\xi = \sum_{\tau=-P}^{P} \left[\sum_{\rho=-P}^{P} a_i R_{x_1 x_1 x_1} (\tau + i, \rho + i) - R_{x_2 x_1 x_1} (\tau, \rho) \right]^2$$
$$= (\mathbf{R}_{x_1 x_1 x_1} \mathbf{A} - \mathbf{R}_{x_2 x_1 x_1})^T (\mathbf{R}_{x_1 x_1 x_1} \mathbf{A} - \mathbf{R}_{x_2 x_1 x_1})$$
(7)

式中,

$$\boldsymbol{R}_{x_{1}x_{1}x_{1}} = \begin{bmatrix} R_{x_{1}x_{1}x_{1}}(0,0) & R_{x_{1}x_{1}x_{1}}(1,1) & \cdots & R_{x_{1}x_{1}x_{1}}(2P,2P) \\ R_{x_{1}x_{1}x_{1}}(-1,-1) & R_{x_{1}x_{1}x_{1}}(0,0) & \cdots & R_{x_{1}x_{1}x_{1}}(2P-1,2P-1) \\ \vdots & \vdots & \ddots & \vdots \\ R_{x_{1}x_{1}x_{1}}(-2P,-2P) & R_{x_{1}x_{1}x_{1}}(-2P+1,-2P+1) & \cdots & R_{x_{1}x_{1}x_{1}}(0,0) \end{bmatrix}$$

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为 $(2P+1) \times (2P+1)$ 维. $A = [a_{-P}, a_{-P+1}, \cdots, a_0, \cdots, a_{P-1}, a_P]^T$ 为 $(2P+1) \times 1$ 维. $R_{x_2x_1x_1} = [R_{x_2x_1x_1}(P, P), R_{x_2x_1x_1}(P-1, P-1), \cdots, R_{x_2x_1x_1}(0, 0), \cdots, R_{x_2x_1x_1}(-P+1, -P+1), R_{x_2x_1x_1}(-P, -P)]^T$ 为 $(2P+1) \times 1$ 维. ξ 的梯度为

$$\nabla = \partial \xi / \partial A = 2(R_{x_1 x_1 x_1}^T R_{x_1 x_1 x_1} A - R_{x_1 x_1 x_1}^T R_{x_2 x_1 x_1})$$
(8)

实际计算中, (7), (8) 式中的三阶累积量 $R_{x_1x_1x_1}$ 及互累积量 $R_{x_2x_1x_1}$ 需要用其估计值 $\hat{R}_{x_1x_1x_1}$, $\hat{R}_{x_2x_1x_1}$ 来代替。梯度类型的自适应算法如下:

$$\xi(k) = \sum_{\tau=-P}^{P} \left[\sum_{\rho=-P}^{P} \hat{a}_{i}(k) \hat{R}_{x_{1}x_{1}x_{1}}(k, \tau+i, \rho+i) - \hat{R}_{x_{2}x_{1}x_{1}}(k, \tau, \rho) \right]^{2}$$
$$= (\hat{R}_{x_{1}x_{1}x_{1}}(k) \hat{A}(k) - \hat{R}_{x_{2}x_{1}x_{1}}(k))^{T} (\hat{R}_{x_{1}x_{1}x_{1}}(k) \hat{A}(k) - \hat{R}_{x_{2}x_{1}x_{1}}(k))$$
(9)

为 k 时刻 ξ 的估计, $\hat{A}(k) = [a_{-P}(k), a_{-P+1}(k+1), \cdots, a_0(k+1), \cdots, a_{P-1}(k+1), a_P(k+1)]^T$, $\hat{R}_{x_1x_1x_1}(k), \hat{R}_{x_2x_1x_1}(k) \in k$ 时刻 $R_{x_1x_1x_1}, R_{x_2x_1x_1}$ 的估计。因此,梯度的估计为

$$\hat{\nabla}(k) = \partial \hat{\xi}(k) / \partial \hat{A}(k) = 2(\hat{R}_{x_1 x_1 x_1}^T(k) \hat{R}_{x_1 x_1 x_1}(k) \hat{A}(k) - \hat{R}_{x_1 x_1 x_1}^T(k) \hat{R}_{x_2 x_1 x_1}(k))$$
(10)

参数迭代公式为

$$\hat{\boldsymbol{A}}(k+1) = \hat{\boldsymbol{A}}(k) - \hat{\boldsymbol{\mu}}(k)\hat{\nabla}(k)$$
(11)

当 0 < $\mu(k)$ < 1/tr{ $\hat{\boldsymbol{R}}_{x_1x_1x_1}^T(k)\hat{\boldsymbol{R}}_{x_1x_1x_1}(k)$ } 时,可保证算法的稳定性。

考虑到这样一个事实,对于形如 (2) 式的信号,当自适应的参数模型时延估计法收敛时, 最佳权系数应为 ^[9],

$$\frac{\alpha \cdot \text{SNR}}{1 + \text{SNR}} [\operatorname{sinc}(i - D_{12}), \ i = -P, -P + 1, \cdots, 0, \cdots, P - 1, P]^T$$
(12)

当衰减系数 $\alpha = 1$,且信噪比较高时, (12)式成为

$$[\operatorname{sinc}(i - D_{12}), \ i = -P, -P + 1, \cdots, 0, \cdots, P - 1, P]^T$$
(13)

依然采用如图 1 所示的结构模型,只是用 [sinc($i - \hat{D}_{12}$), $i = -P, \dots, P$] 替换权系数 a_i , \hat{D}_{12} 是瞬时时延估计。这样,所有的权系数都将由 \hat{D}_{12} 决定,最佳准则成为

$$\xi(k) = [\mathbf{R}_{x_1 x_1 x_1}(k) \mathbf{A}(k) - \mathbf{R}_{x_2 x_1 x_1}(k)]^T [\mathbf{R}_{x_1 x_1 x_1}(k) \mathbf{A}(k) - \mathbf{R}_{x_2 x_1 x_1}(k)]$$
(14)

式中 $A = [\operatorname{sinc}(i - \hat{D}_{12}), i = -P, \dots, P]$, 有约束自适应时延直接估计算法成为

$$\hat{D}_{12}(k+1) = \hat{D}_{12}(k) - \mu \frac{\partial \xi(k)}{\partial \hat{D}_{12}(k)} = \hat{D}_{12}(k) + 2\mu C(k)$$
(15)

式中 $C(k) = (\hat{\boldsymbol{R}}_{x_1x_1x_1}^T \hat{\boldsymbol{R}}_{x_1x_1x_1} \hat{\boldsymbol{A}} - \hat{\boldsymbol{R}}_{x_1x_1x_1}^T \hat{\boldsymbol{R}}_{x_2x_1x_1})^T \hat{\boldsymbol{F}}(k), \hat{\boldsymbol{F}}(k) = [f(i-\hat{D}_{12}(k)), i = -P, \cdots, P]^T,$ $f(v) = [\cos(\pi v) - \operatorname{sinc}(v)]/v, \mu$ 是一收敛因子, 控制算法的稳定性。

3 算法收敛性分析

通常假定 $\{s(k)\}$ 与 $\hat{D}_{12}(k)$ 不相关,对 (15) 式两边取数学期望,

$$E\{\hat{D}_{12}(k+1)\} = E\{\hat{D}_{12}(k)\} + 2\mu E\{(\hat{R}_{x_{1}x_{1}x_{1}}^{T}\hat{R}_{x_{1}x_{1}x_{1}}\hat{A} - \hat{R}_{x_{1}x_{1}x_{1}}\hat{R}_{x_{2}x_{1}x_{1}})^{T}\hat{F}(k)\}$$

$$= E\{\hat{D}_{12}(k)\}$$

$$+ 2\mu E\left\{\sum_{\tau=-P}^{P}\left[\left(\sum_{i=-P}^{P}\operatorname{sinc}(i-\hat{D}_{12}(k))\hat{R}_{x_{1}x_{1}x_{1}}(\tau+i,\tau+i) - \hat{R}_{x_{2}x_{1}x_{1}}(\tau,\tau)\right)\right)$$

$$\times \left(\sum_{j=-P}^{P}f(j-\hat{D}_{12}(k))\hat{R}_{x_{1}x_{1}x_{1}}(\tau+j,\tau+j)\right)\right]\right\}$$

$$= E\{\hat{D}_{12}(k)\} + 2\mu E\left\{\sum_{\tau=-P}^{P}\left[\left(\sum_{i=-P}^{P}\operatorname{sinc}(i-\hat{D}_{12}(k))\hat{R}_{x_{1}x_{1}x_{1}}(\tau+i,\tau+i)\right)\right)$$

$$\times \left(\sum_{j=-P}^{P}f(j-\hat{D}_{12}(k))\hat{R}_{x_{1}x_{1}x_{1}}(\tau+j,\tau+j)\right)\right]\right\}$$

$$- 2\mu E\left\{\sum_{\tau=-P}^{P}\left[\left(\sum_{i=-\infty}^{\infty}\operatorname{sinc}(i-D_{12})\hat{R}_{x_{1}x_{1}x_{1}}(\tau+i,\tau+i)\right)\right)$$

$$\times \left(\sum_{j=-P}^{P}f(j-\hat{D}_{12}(k))\hat{R}_{x_{1}x_{1}x_{1}}(\tau+j,\tau+j)\right)\right]\right\}$$
(16a)

上式中,后两项只有i=j时才存在,因此有

$$E\{\hat{D}_{12}(k+1)\} = E\{\hat{D}_{12}(k)\} \\ + 2\mu E\left\{\sum_{\tau=-P}^{P}\left[\sum_{i=-P}^{P}\operatorname{sinc}(i-\hat{D}_{12}(k))\cdot f(i-\hat{D}_{12}(k))\cdot \hat{R}_{x_{1}x_{1}x_{1}}^{2}(\tau+i,\tau+i)\right]\right\} \\ - 2\mu E\left\{\sum_{\tau=-P}^{P}\left[\sum_{i=-P}^{P}\operatorname{sinc}(i-D_{12}))\cdot f(i-\hat{D}_{12}(k))\cdot \hat{R}_{x_{1}x_{1}x_{1}}^{2}(\tau+i,\tau+i)\right]\right\}$$
(16b)

对上式交换求和与取期望的次序,有

$$E\{\hat{D}_{12}(k+1)\} = E\{\hat{D}_{12}(k)\}$$

$$+ 2\mu \left\{ \sum_{i=-P}^{P} \left[E(\operatorname{sinc}(i - \hat{D}_{12}(k))f(i - \hat{D}_{12}(k)) \cdot E\left(\sum_{\tau=-P}^{P} \hat{R}_{x_{1}x_{1}x_{1}}^{2}(\tau + i, \tau + i)\right) \right] \right\}$$

$$- 2\mu \left\{ \sum_{i=-P}^{P} \left[E(\operatorname{sinc}(i - D_{12})f(i - \hat{D}_{12}(k)) \cdot E\left(\sum_{\tau=-P}^{P} \hat{R}_{x_{1}x_{1}x_{1}}^{2}(\tau + i, \tau + i)\right) \right] \right\}$$

$$= E\{\hat{D}_{12}(k)\}$$

$$+ 2\mu \frac{\operatorname{tr}(\hat{R}_{s_{1}s_{1}s_{1}s_{1}}^{T}\hat{R}_{s_{1}s_{1}s_{1}})}{2P + 1} E\left\{ \sum_{i=-P}^{P} (\operatorname{sinc}(i - \hat{D}_{12}(k))f(i - \hat{D}_{12}(k))\right\}$$

$$- 2\mu \frac{\operatorname{tr}(\hat{R}_{s_{1}s_{1}s_{1}s_{1}}^{T}\hat{R}_{s_{1}s_{1}s_{1}})}{2P + 1} E\left\{ \sum_{i=-P}^{P} (\operatorname{sinc}(i - D_{12}))f(i - \hat{D}_{12}(k))\right\}$$

$$(16c)$$

由于, $f(k - \hat{D}_{12}(k)) = \sum_{i=-\infty}^{\infty} \operatorname{sinc}(i - \hat{D}_{12}(k))f(k - i)$, 且 f(0) = 0, 那么有

$$E\{\hat{D}_{12}(k+1)\} = E\{\hat{D}_{12}(k)\} - 2\mu \frac{\operatorname{tr}(\hat{\boldsymbol{R}}_{s_1s_1s_1}^{T}\hat{\boldsymbol{R}}_{s_1s_1s_1})}{2P+1} E\{f(D_{12} - \hat{D}_{12}(k))\}$$
(16d)

对 $f(D_{12} - \hat{D}_{12}(k))$ 进行 Taylor 展开, 忽略高阶量, 可得

$$f(D_{12} - \hat{D}_{12}(k)) \approx \frac{1}{D_{12} - \hat{D}_{12}(k)} - \frac{\pi^2}{2} (D_{12} - \hat{D}_{12}(k)) - \frac{1}{D_{12} - \hat{D}_{12}(k)} + \frac{\pi^2}{6} (D_{12} - \hat{D}_{12}(k))$$
$$= \frac{\pi^2}{3} (\hat{D}_{12}(k) - D_{12})$$
(16e)

可有

$$E\{\hat{D}_{12}(k+1)\} = E\{\hat{D}_{12}(k)\} - 2\mu \frac{\operatorname{tr}(\hat{R}_{s_{1}s_{1}s_{1}s_{1}}^{T}\hat{R}_{s_{1}s_{1}s_{1}})}{2P+1} \frac{\pi^{2}}{3} E\{\hat{D}_{12}(k) - D_{12}\}$$
$$= D_{12} + E\{\hat{D}_{12}(k) - D_{12}\} \left[1 - \frac{2}{3}\mu\pi^{2} \frac{\operatorname{tr}(\hat{R}_{s_{1}s_{1}s_{1}}^{T}\hat{R}_{s_{1}s_{1}s_{1}})}{2P+1}\right]$$
$$= D_{12} + E\{\hat{D}_{12}(0) - D_{12}\} \left[1 - \frac{2}{3}\mu\pi^{2} \frac{\operatorname{tr}(\hat{R}_{s_{1}s_{1}s_{1}}^{T}\hat{R}_{s_{1}s_{1}s_{1}})}{2P+1}\right]^{k+1}$$
(17)

因此,只要下面条件满足时,可保证算法收敛:

$$0 < \mu(k) < \frac{3(2P+1)}{\pi^2 \operatorname{tr}(\boldsymbol{R}_{s_1 s_1 s_1}^T \boldsymbol{R}_{s_1 s_1 s_1})}$$
(18)

4 仿真结果

直升机噪声是明显的非高斯信号,本节将上述算法应用于空中运动目标的时变时延估计,以 实测的直升机噪声信号为例进行仿真,信号的采样频率为5kHz.首先对直升机的方位估计可建 立如下模型.采用十字形声传感器阵列,4个阵元的坐标分别为 $S_1(D/2,0,0)$, $S_2(0,D/2,0)$, $S_3(-D/2,0,0)$, $S_4(0,-D/2,0)$,目标声源T的坐标为(x,y,z),D为阵元间距,这里D = 2m. 限于篇幅,本文以阵元 S_1 , S_3 接收信号时延 τ_{13} 的估计结果为例加以说明(图 2).

当空间噪声相关时, 两路接收到的离散时间信号可假设为如下形式:

$$x_{1}(k) = s(k) + n_{1}(k) x_{2}(k) = s(k - D_{12}(k)) + n_{1}(k - D_{n})$$
(19)

仿真中,假定(19)式中 $D_n = -4$.并假定如下典型飞行状况,飞行速度V = 50m/s,高度h = 50m,直升机从水平距离 300m 处飞行至反方向 100m 处,即从要求开始跟踪的距离 处飞行到通过攻击区域。假设当直升机位于水平距离 300m 处时的信噪比为 5dB,仿真模型同 时考虑到随着目标与定位系统接近时,信噪比的变化。图 3 给出了该飞行条件下的理论值,图 4—图 6 分别给出了基于二阶统计量的算法、三阶累量域的无约束和有约束的自适应算法进行 估计的一次估计结果。基于累量域的收敛因子分别取为 $\mu(k) = 0.8/\text{tr}\{(\hat{R}_{x_1x_1x_1}(k)\hat{R}_{x_1x_1x_1}(k)\},$



图 2 (a) 十字阵定位示意图 (b) 十字阵阵形示意图

 $\mu(k) = 5/tr\{(\hat{R}_{x_1x_1x_1}^T(k)\hat{R}_{x_1x_1x_1}(k)\}$. 二阶统计量的计算长度为 64, 三阶累积量的计算取长 度为 128×6,数据分段时,相邻两段的重叠率为 20%.



图 5 时延 τ13 无约束三阶累量算法估计结果

图 6 时延 τ13 有约束三阶累量算法估计结果

0

2

0

2

5结论

仿真表明:当空间噪声相关且信噪比较低时,基于二阶统计量的算法不再给出正确的时延估计结果,只有当信噪比较高时,算法才能给出正确的结果。三阶累量域的算法,能在空间相关高斯噪声条件下,跟踪信号时延的变化,但需要更长的数据长度,及更大的计算量。有约束自适应时延估计算法明显优于无约束的算法。在该条件下,可以较好地跟踪时延变化。

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A KIND OF TIME VARIANT TIME DELAY ESTIMATION METHOD UNDER SPATIALLY CORRELATED GAUSSIAN NOISE

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Abstract When the noises are spatially correlated, the time delay methods based on second order statistic will be useless. In this paper a kind of adaptive time delay estimation method based on third order statistic is provided, and its convergence is analyzed. Finally, simulation results demonstrate that this approach can effectively depress Gaussian correlated noise.

Key words Time delay estimation, Third order statistic

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